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METEOSAT: PCM TRANSMISSION OF IMAGES

N. Darche

Translation of "Meteosat Transmission PCM des Images: Definition du Format ainsi que de la Strategie du Synchronisateur Secondaire".

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Solar Equipment Division
Studies and Projects Department

METEOSAT
PCM TRANSMISSION OF IMAGES

Format definition as well as the secondary synchronizer strategy.

This report is a follow-up to Report No. 533/CB/ES/E.

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BRETIGNY, October 31, 1970
No. 567/CB/ES/E

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N. DARCHE

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INTRODUCTION

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In a PCM transmission of information, the sequence of the transmitted message is divided into formats (or subcycles), each format being itself divided into a certain number of cycles. A cycle, in turn, is composed of words and each word can be divided into syllables.

In the application of METEOSAT to image telemetry and in the hypothesis of a continuous transmission, the format corresponds to an image, each cycle to a line, each word to a point on the ground. The division into syllables corresponds to the distribution of the information between visible channel(s) and infrared channel(s).

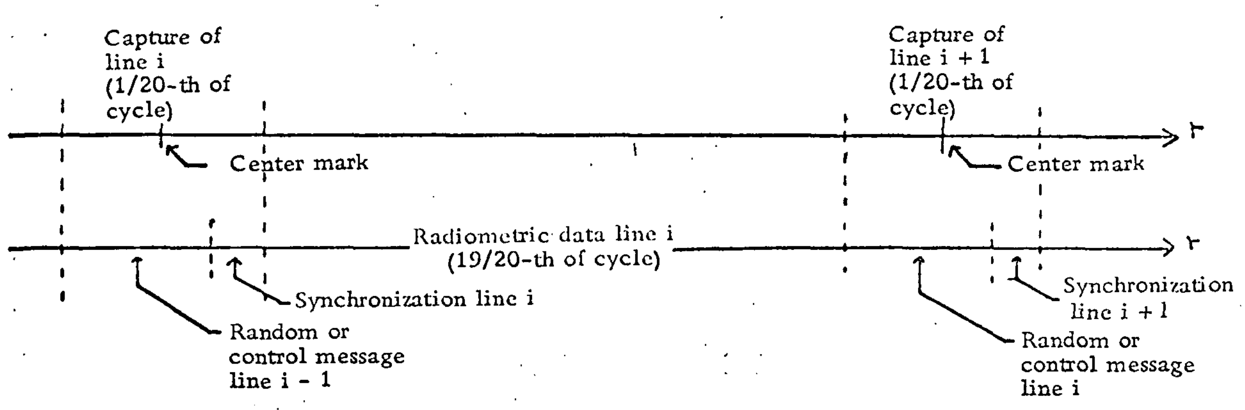
Since the transmission is a series transmission, it is necessary to be able to mark the beginning of a format as well as the beginning of each cycle. This is the reason why one intersperses cycle and subcycle synchronization words with the message words. Then, upon receiving the information, the location of these particular words has to be detected; this is the role of the secondary synchronizer.

HYPOTHESES

Here we are interested first of all in the case of a continuous transmission with only one stored line.

In other words the radiometric data are taken during 1/20th of a cycle and placed in a memory; then they are retransmitted to the ground during the remaining 19/20th of the cycle. In order not to increase the transmission time uselessly, the synchronization words of the line image would be transmitted before the end of the data taking. On the other hand, in order not to interrupt the transmission between two lines, a random signal is transmitted to the ground during the remainder of the data-taking period.

The timing diagram is as follows:



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*Numbers in the margin indicate pagination in the foreign text.

FIRST PART - FORMAT DEFINITION /4

Given the number of points contained in an image, the PCM transmission format for METEOSAT cannot be a standard format. None of the present-day specifications on the subject of the cycle synchronization word length, the number of words per cycle and the number of cycles per subcycle is applicable to the case at hand.

The format used for METEOSAT is shown in Fig. 1.

It has already been shown (Report No. 533 CB/ES/E) that is is sufficient to place a single synchronization word at the beginning of a line to ensure message acquisition. Consequently a cycle of the format corresponds to a line of the image, while a subcycle corresponds to an image.

I. CYCLE SYNCHRONIZATION

The cycle synchronization word is a P.N. code, 63 or 127 bits long. The final choice of the length can be made only when the number of visible and infrared channels, the number of quantification bits and the number of points per line have been defined.

The line length varies, as a result, from 27,500 bits for 1 visible channel, 1 infrared channel and 2000 points per line, through 70,000 bits for 1 infrared channel, 2 visible channels and 2500 points per line, to 121,000 bits for 2 visible channels, 2 infrared channels and 3200 points per line.

The 63-bit length for the synchronization code is valid for a line of the order of 70,000 bits. The 127-bit code is necessary if one wants to maintain a P.N. code.

A METEO image, on the other hand, is stationary in nature. To compensate for this, the cycle synchronization code is alternated from one cycle to another.

1) 63-bit code /6

This code is obtained by means of 6 flip-flops mounted in a shift register that circles back on itself by a reaction of flip-flops 1 and 6.

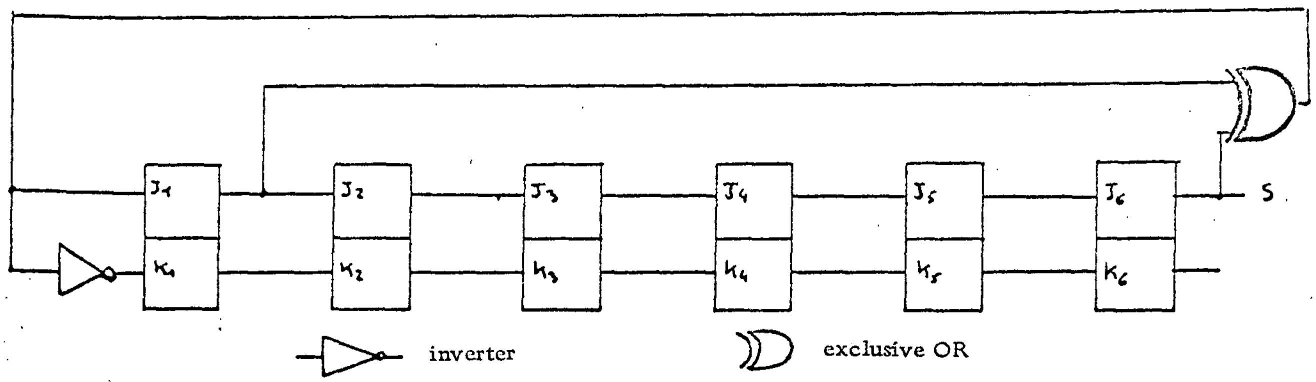
The code obtained in this fashion depends on the initial state of the flip-flops. The code choice has been determined by the detection probability of the code in the search phase of the secondary synchronization (§ 11.2, Report No. 533 CB/ES/E) and by the probability of detecting the code in the locking phase (Appendix to this report).

Initial state: all flip-flops are in 1 and the code obtained is:

111111010101100110111011010010011100010111100101000110000100000

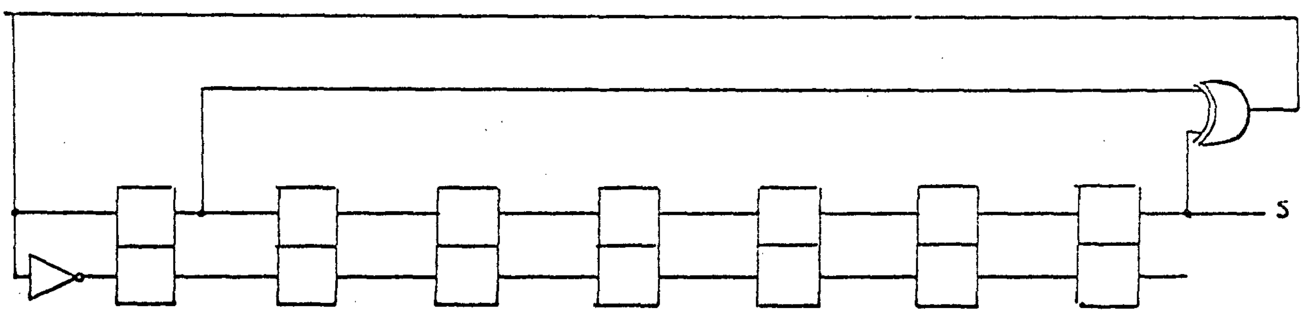
Cycle synchro., 63 or 127 bits S	Cycle ident., 33 or 36 bits 1	Radiometric information 1st line of image	Control information + PN code
$\bar{S}$	2	Radiometric information 2nd line of image	Control information + PN code
S	3	Radiometric information 3rd line of image	Control information + PN code
$\bar{S}$	4	Radiometric information 4th line of image	Control information + PN code
$\bar{S}$	L	Radiometric information last line of image	Control information + PN code

Figure 1.



2) <u>127-bit code</u>

This code is obtained like the previous one with 7 flip-flops, the reaction being accomplished on the flip-flops 1 and 7. For the initial state all flip-flops are in 1.



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II. SUBCYCLE SYNCHRONIZATION /7

The subcycle synchronization is accomplished by a counting of the lines. In pure binary form, such a counting necessitates 11 bits if the number of lines is less than or equal to 2048, 12 bits otherwise.

Although the bit error probability anticipated for METEOSAT would be 10^{-6}, in order that the image be usable, it is desirable that the system function up to a probability of 10^{-2} so that synchronization not be lost during perturbations.

When $p = 10^{-2}$, the probability that there will be e errors among 12 bits is equal to:

$$P(e) = C_{12}^{e} \; p^{e} (1 - p)^{12 - e}$$

The probability that the number of one line would be incorrect is the probability that there would be a number of errors greater than or equal to 1. P_F would be this probability:

$$P_F = \sum_{e=1}^{12} C_{12}^{e}\, p^{e}\, (1-p)^{12-e} = 1 - C_{12}^{o}\, p^{o}(1-p)^{12} = 1 - (1-10^{-2})^{12} = 1.14\ 10^{-1}$$

If L is the number of lines in an image and l is the number of lines for which the counting is incorrect:

$$p(\ell) = C_L^{\ell}\, (P_F)^{\ell}\, (1 - P_F)^{L - \ell}$$

The average value of l is then $< l > = LP_F$. If the image contains 3200 lines, $< l > = 365$. Such a result is unacceptable. It is therefore necessary to correct a certain number of errors. The use of a correction code for 2 errors would be too unwieldly to place in operation. Two solutions can then be considered:

A 1-error correction and a 2-error detection code.

Pure binary counting, repeated 3 times.

1) <u>1-error correction and 2-error detection code</u> /8

For 12 bits of information, one can use a systematic code with a length of n + 1, such that n = m + k, where m is the number of information bits, and k is the number of control bits.

A systematic code is such as $2^k \geq n + 1$ (Ref. 1). For m = 12 one then finds k = 5 and n = 17. The resulting code is correct if the code transmitted contains 0 or 1 error, i.e., with a probability:

$$P = C_{18}^{o}\ p^{o}\ (1-p)^{18} + C_{18}^{1}\ p\ (1-p)^{17}$$

$$= (1 - 10^{-2})^{18} + 0.18\ (1 - 10^{-2})^{17}$$

$$= 1.17\ (1 - 10^{-2})^{17} = 9.86\ 10^{-1}$$

The probability that it contains 2 errors is equal to:

$$P(2) = C_{18}^{2} \, p^{2}(1 - p)^{16} = 1.30 \; 10^{-3}$$

2) <u>Pure binary counting, repeated 3 times</u>

Upon reception a majority decision is made for each bit. Since a bit is repeated 3 times, it would be correct after detection if it was not incorrect more than once before detection.

The probability that a bit will be correct is then equal to:

$$P_b = C_3^0 \, p^0 \, (1 - p)^3 + C_3^1 \, p \, (1 - p)^2$$

$$P_b = (1 - p)^3 + 3 \, p \, (1 - p)^2 = 1 - 3p^2 + 2p^3$$

The line counting will be correct after detection if all the bits are correct, i.e., with a probability

$$P = (1 - 3p^2 + 2p^3)^{12}$$

$$P \approx 1 - 3.6 \; 10^{-3} \quad \text{for} \quad p \doteq 10^{-2}$$

This method is more efficient than the previous one; therefore this is the one that will be adopted.

III. <u>RADIOMETRIC INFORMATION</u>

This part of the cycle is formed of P words of constant length, where P is the number of points per line of the image. /9

Each word is subdivided into a certain number of syllables in accordance with the distribution of the visible and infrared channels. For example:

1 VIS and 1 IR channel - Each word consists of one syllable of 5 bits for the visible information and one syllable of 8 bits for the infrared information.

2 VIS channels and 1 IR channel - 2 syllables of 5 bits for a first sampling of the visible channels, 1 syllable of 8 bits for the IR channel, 2 syllables of 5 bits for a second sampling of the visible channels.

If b is the number of bits per word, the length of this part of the cycle will be: $b \times P$ bits.

IV. CONTROL INFORMATION, PN CODE

During the remainder of the cycle, which corresponds to a little less than one-twentieth of the cycle (§I), the number of available bits is sufficient for transmitting the control information. If the control information does not occupy all of this last part of the cycle, it is necessary to transmit a PN code in order to maintain a continuous transmission until the appearance of the synchronization word of the following cycle.

Because of the fact that the location of the synchronization pulse given by the spin clock possesses a certain "jitter" (Ref. 3), the difference between two consecutive synchronization words of the format is not constant, which is reflected in the length of the last part of the cycle.

The length of the block formed by the control information and the PN code is equal to:

$$\frac{b \times P}{19} - \text{(length of cycle synchronization word)} - \text{(length of cycle identification word)} \pm \Delta L$$

where ΔL is equal to the amplitude of the error of the location of the synchronization pulse.

SECOND PART - STRATEGY PROPOSED FOR THE SECONDARY SYNCHRONIZER /10

In a PCM telemetry acquisition system, the secondary synchronizer follows the primary synchronizer, designed to retrieve the pattern of the bits and to determine, for each period of this pattern, if the bit transmitted is a "1" or a "0". The secondary synchronizer then receives a train of bits, synchronized by the pattern provided by the primary synchronizer.

In this train of bits, the secondary synchronizer must first mark the configuration of bits corresponding to the cycle synchronization word. When this position is detected, the 33 (or 36) subsequent bits provide the cycle number.

A. CYCLE SYNCHRONIZATION (DESCRIPTION OF OPERATION)

1) General case

In a classical secondary synchronizer the cycle synchronization is accomplished in three phases:

Search phase

If the synchronization code has N bits, one compares, during the period of 1 cycle, each received configuration of N consecutive bits with the expected code. The configuration nearest to that expected then permits the code location to be marked.

Control phase

During C consecutive cycles one tries to verify that the detection has been correct. There is confirmation if the position detected for the code during the search phase is again the best for the cycle examined, otherwise there is invalidation. If, during the C cycles examined during the control phase, the number of invalidations remains less than a value I1, the system passes into the locking phase. Otherwise, it returns to the search phase.

Locking phase

In the locking phase the code position is assumed to have been determined correctly. It is then possible to authorize more errors with respect to the ideal configuration than in the other two phases. There is an invalidation /11
if the number of errors exceeds the threshold (threshold = number of errors authorized). If there are more than I2 successive invalidations, the system returns to the control phase.

2) Application to METEOSAT

The control phase is suppressed for METEOSAT because the search phase is sufficient. The linking of the two phases - Search and Locking - is shown in Fig. 2.

2.1) Search phase (Fig. 3)

Since the code has a length of N bits, the message bits are examined in groups of N, and by successive one-bit shifts, all possible code positions in the received message are examined.

On the other hand, the cycle synchronization code is alternated from one cycle to the next and the search code has two possible configurations. Consequently, for each position, a correlator determines the number of errors E1 with respect to the S code and the number of errors E2 with respect to the complement $\bar{S}$. The lower value is retained (for example E1) and compared to a previously stored value, the threshold E. If E1 is less than the threshold, this position is more probable than the previously detected one; the threshold is then modified to take account of the value E1 and one concludes that this is the position of the transmitted synchronization code (the bit counter is set back to zero). If E1 is greater than or equal to the threshold, there is a confirmation of the fact that the previously detected code is valid, the threshold value is retained and the bit counter is incremented by one. The search phase is terminated when the bit counter reaches the value M (the number of bits in the cycle).

The code detected at the end of the search phase is then correct if, when the received code has E errors with respect to the transmitted code, the positions previously examined exhibit more than E errors with respect to the transmitted code or to its complement, and if the positions examined during the remainder of the cycle exhibit E errors or more with respect to the transmitted code and to its complement.

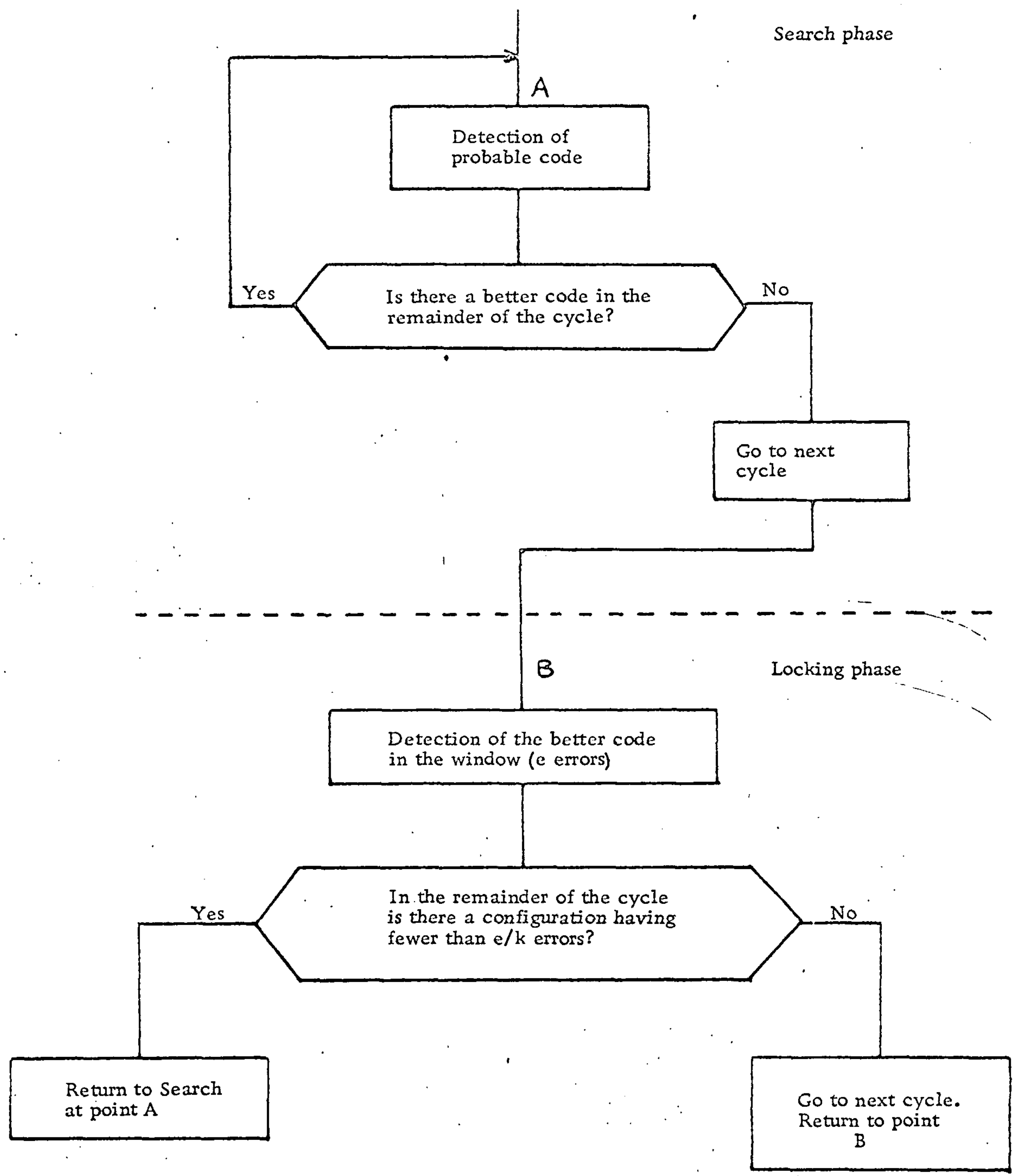


Figure 2.

Cycle synchronization.

Phase linking.

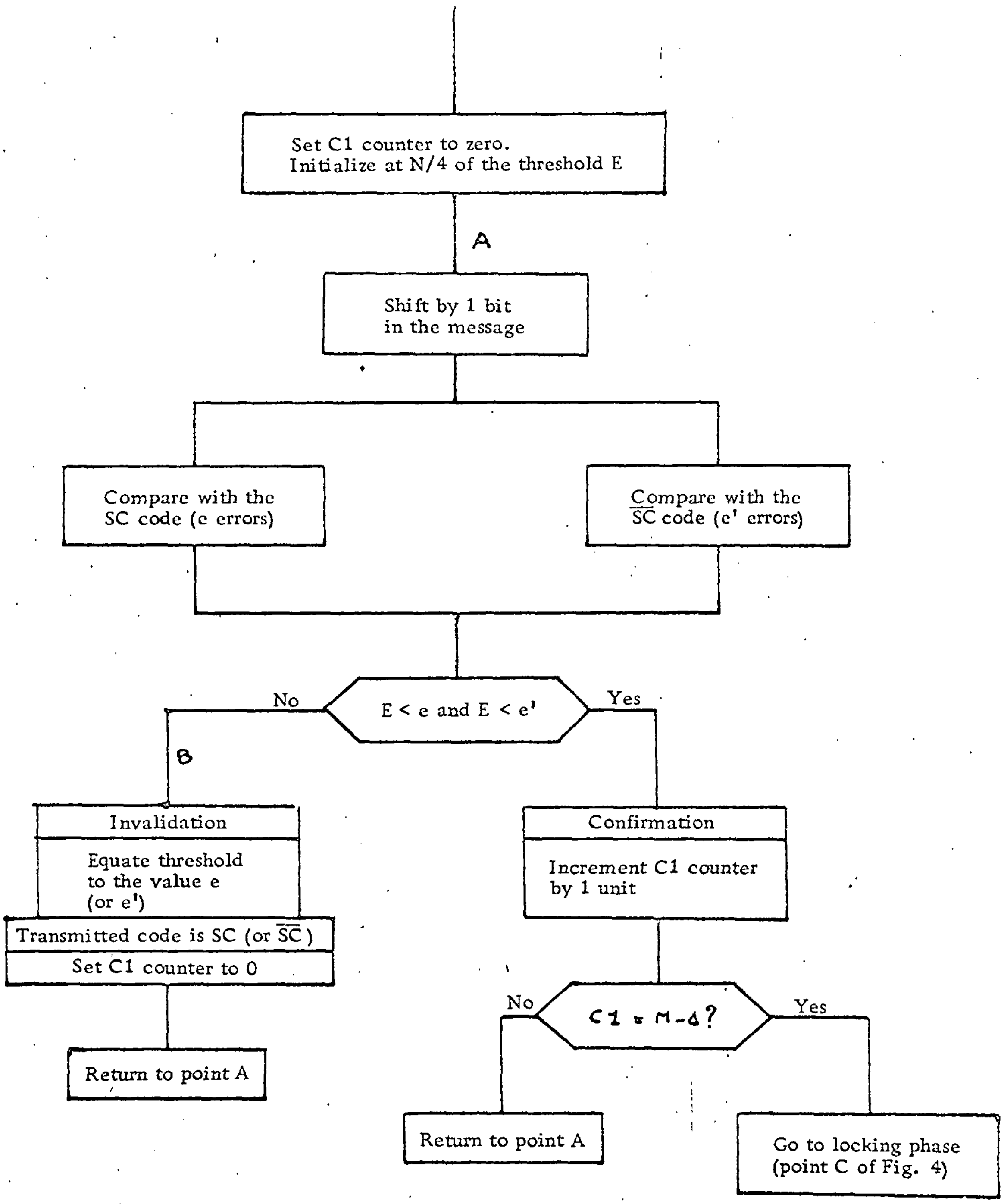


Figure 3.

Cycle synchronization.

SEARCH phase.

It is advisable, on the other hand, that the number of errors detected should be less than N/4 in order that the corresponding position can be assumed to be that of the transmitted code (this is to avoid a break in the absence of the signal).

The probability of detecting the code in its correct position, for an alternated code, is given by: /14

$$P_d = \sum_{e=o}^{N/4} C_N^e \, p^e (1-p)^{N-e} \left[\left(\tfrac{1}{2}\right)^N \sum_{j=e+1}^{N-e-1} C_N^j \right]^{M-1} \qquad \text{(Ref. 2)}$$

where p is the bit error probability. For $p = 10^{-2}$ and $N = 63$ one finds that $p_d = 1 - 1.15 \times 10^{-10}$. Similarly the probability that this position will be confirmed during the remainder of the cycle is:

$$Pc = \sum_{e=o}^{N/4} C_N^e \, p^e (1-p)^{N-e} \left[\left(\tfrac{1}{2}\right)^N \sum_{j=e+1}^{N-e-1} C_N^j \right]^{M-1} \left[\left(\tfrac{1}{2}\right)^N \sum_{j=e}^{N-e} C_N^j \right]^{M-1}$$

$$Pc > \sum_{e=o}^{N/4} C_N^e \, p^e (1-p)^{N-e} \left[\left(\tfrac{1}{2}\right)^N \sum_{j=e+1}^{N-e-1} C_N^j \right]^{2M-2}$$

$$Pc > \left[\sum_{e=o}^{N/4} C_N^e \, p^e (1-p)^{N-e} \left[\left(\tfrac{1}{2}\right)^N \sum_{j=e+1}^{N-e-1} C_N^j \right]^{M-1} \right]^2 \qquad \text{(verified by calculation)}$$

$$Pc > Pd^2$$

$$Pc > 1 - 2{,}3 \; 10^{-10}$$

For such probabilities it is unnecessary to provide a control phase. The end of the search phase will then join directly to the locking phase.

2.2) Locking phase (Fig. 4) /15

Opening of a window about the theoretical position of the code

When the number of bits per cycle is known with certainty and when the code position in a cycle has been ascertained, it suffices to count the number of bits in order to know the position of the synchronization code of the next cycle.

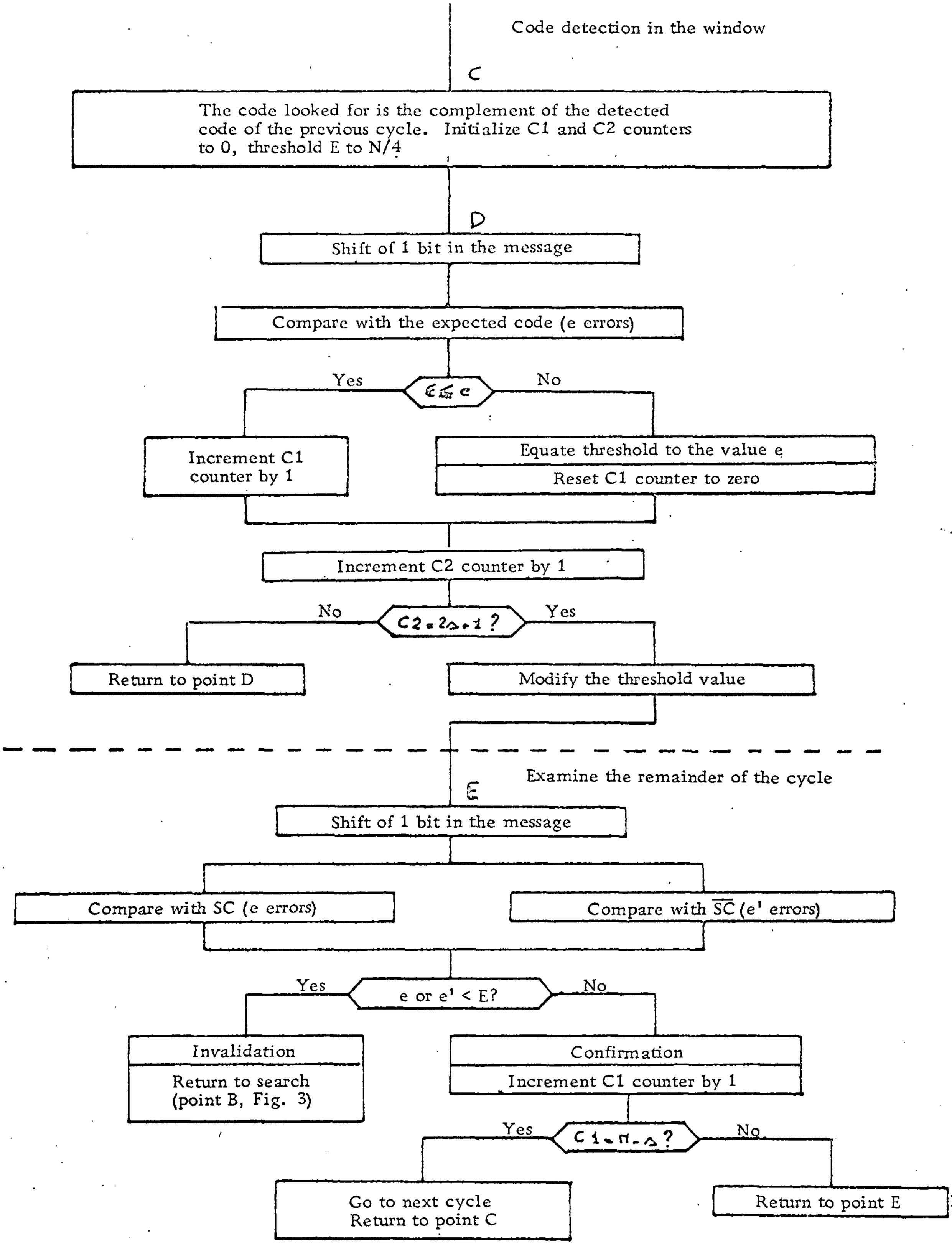


Figure 4.

Cycle synchronization.

locking phase.

Within the METEOSAT system, independently of the bit shifts that can be introduced by the primary synchronizer, the number of bits in a cycle is variable because of the uncertainty in the position of the synchronization pulses delivered by the spin clock. Instead of looking for the code at a fixed position with respect to the code of the preceding cycle, the search then encompasses several bits before and several bits after. To do this, the bit counter does not count up to M (the theoretical average number of bits per cycle) but up to $M - \Delta$ (Δ is the width of the window opened about the theoretical position of the code, where Δ is greater than or equal to the maximum value of the variation of the length of one cycle). The detection of the code of the following cycle begins at this moment and lasts during $2\Delta + 1$ successive shifts. Since the code is alternated from one cycle to the next, if in cycle n it is the S code which has been transmitted, one looks for the code of the $(n + 1)$ cycle at the position, among $2\Delta + 1$ possible positions, which has the least errors with respect to the complement of S, i.e., $\bar{S}$. It is this position which will be established to be the code position. After this position the bit counter is reset to 0 and the number of errors obtained, e, provides the error threshold for examining the remainder of the cycle, $E = e/k$, where k is set at a value of 2.

Confirmation over the remainder of the cycle

Although the position found for the code might be assumed correct, one continues to examine, in succession, all positions in the cycle, with the bit counter being incremented up to the value $M - \Delta$. For each position the comparison is made with the received synchronization code (e errors) and with its complement (e' errors). If e or e' is less than E, there is an invalidation and a return to the search phase. If no invalidation remains during the remainder of the cycle, the secondary synchronizer is kept in the locking phase for the following cycle.

B. CYCLE SYNCHRONIZATION (QUANTITATIVE STUDY /17

I. Search Phase

The search phase has been studied in the report cited in Ref. 2.

II. Locking Phase

We are interested in the case where the Locking phase is begun after a correct detection of the synchronization code of the preceding cycle.

1) Code detection in the window

The $2\Delta + 1$ positions in the window are examined in succession and the first of these having the fewest errors is retained as the code position.

Detection occurs at a position x ($x \in [-\Delta, +\Delta]$) if, for e errors at this posiposition with respect to the expected code, there are more than e errors at the preceding positions and e errors or more at the subsequent positions, with the comparison always being made with the expected code.

The study has been made in the case where the threshold is not limited in the Locking phase, with the results being valid if the error threshold is limited to N/4 as in the Search phase to avoid remaining locked if a probable code has been detected in the absence of a signal.

1.1) Random hypothesis

From the very first it is assumed that the configurations other than that of the code are random configurations.

If PV(x) represents the probability of making a correct code detection if the latter is in the position x in the window, PV(x) is expressed by:

$$PV(x) = \sum_{e=0}^{N} C_N^e \, p^e (1-p)^{N-e} \left[1-(\tfrac{1}{2})^N \sum_{j=0}^{e} C_N^j\right]^{\Delta+x} \left[1-(\tfrac{1}{2})^N \sum_{j=0}^{e-1} C_N^j\right]^{\Delta-x}$$

where $\Delta + x$ is the number of positions examined before the code, $\Delta - x$ is the number of positions examined after the code, p is the bit error probability, and N is the length of the code.

The different values of PV(x) as a function of x and of Δ are shown on the curves of Figs. 5 and 6 for N = 63 where $PF_x = 1 - PV(x)$. /18

1.2) Real case

Among the positions other than the code, two cases are distinguished in real life: the strictly random positions and those which encompass a part of the code (Ref. 2).

Consequently, among the $\Delta + x$ positions preceding the code, one distinguishes:

b1 positions partially encompassing the code
$\beta 1$ random positions
$$b1 + \beta 1 = \Delta + x$$

Similarly, among the $\Delta - x$ positions which follow the code, one distinguishes:

b2 positions partially encompassing the code
$\beta 2$ random positions
$$b2 + \beta 2 = \Delta - x$$

b1 and b2 fall between 0 and N - 1.

If $\Delta + x < N - 1$, $b1 = \Delta + x$, if $\Delta + x \geq N - 1$, $b1 = N - 1$.

If $\Delta - x < N - 1$, $b2 = \Delta - x$, if $\Delta - x \geq N - 1$, $b2 = N - 1$.

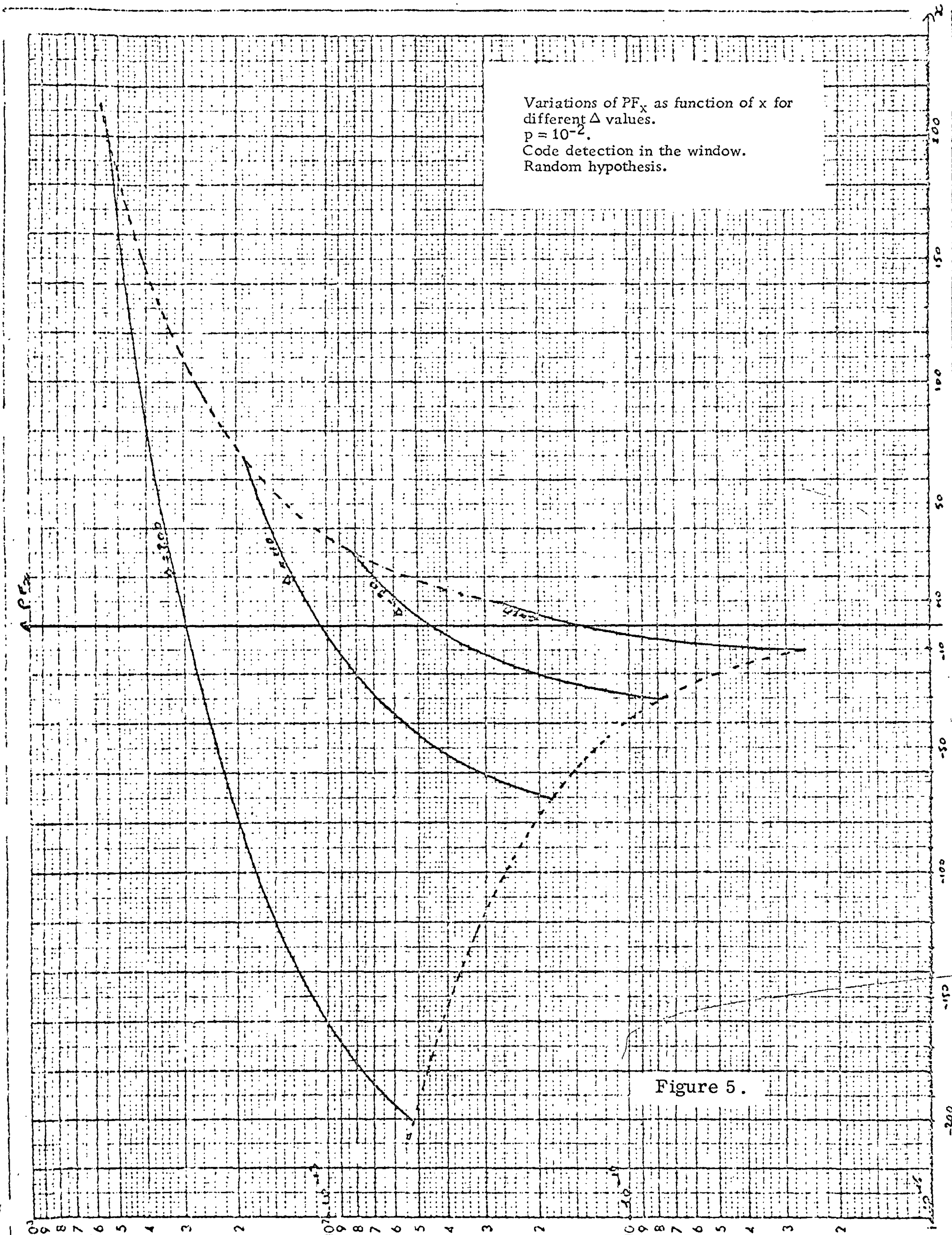

Variations of PF_x as function of x for different Δ values.
$p = 10^{-2}$.
Code detection in the window.
Random hypothesis.

Figure 5.

"Centre National d'Etudes Spatiales	Variations of PF_x as function of x for different Δ values $p = 10^{-6}$	Code detection in the window Random hypothesis

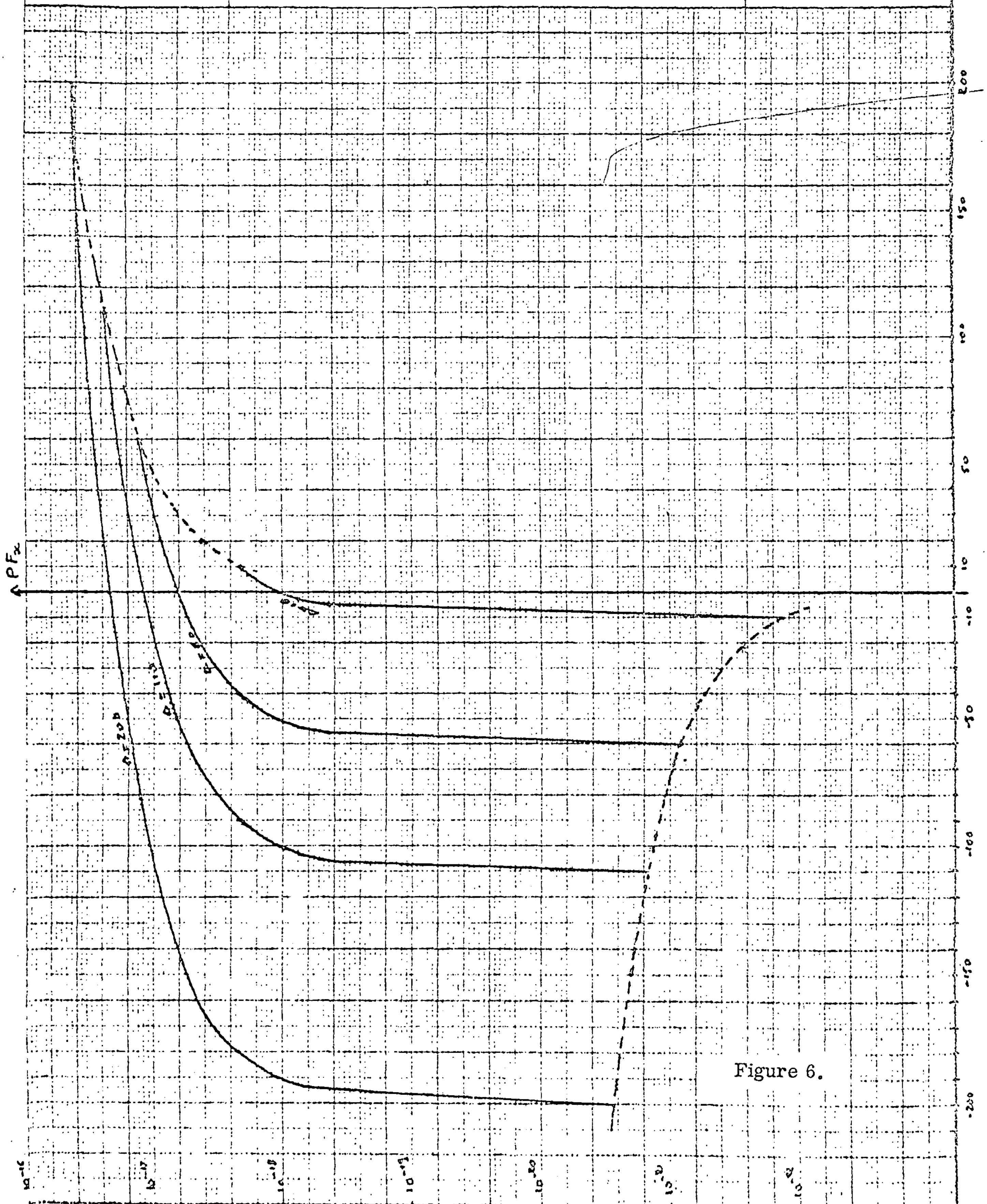


Figure 6.

We designate by m1 the values of the possible overlappings for the positions which precede the code and by m2 the values for the positions which follow the code.

$$m1 \in [N - b1 - 1,\ N - 1]$$

$$m2 \in [N - b2 - 1,\ N - 1]$$

The probability of making a correct detection is then expressed by: /21

$$\boxed{PV(x) = \sum_{e=0}^{N} C_N^e\, p^e (1-p)^{N-e} \left[1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e} C_N^j\right]^{\beta 1} \left[1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e-1} C_N^j\right]^{\beta 2} \prod_{m=N-b1-1}^{N-1} (1-Hm(e)) \prod_{m=N-b2-1}^{N-1} (1-Hm(e-1))}$$

with $$Hm(e) = \left(\tfrac{1}{2}\right)^b \sum_{i=Max(0,c-e)}^{c} C_c^i\, p^i (1-p)^{c-i} \sum_{j=0}^{Min(m-c,\, e-c+i)} C_{m-c}^j\, p^j (1-p)^{m-c-j} \sum_{\beta=0}^{Min(e-c+i-j,\, b)} C_b^\beta$$

where b = N - m and where e denotes the number of bits in disagreement, among the m overlapping bits, between the position being considered and the code.

By writing:

$$Pr(e) = C_N^e\, p^e\, (1-p)^{N-e}$$

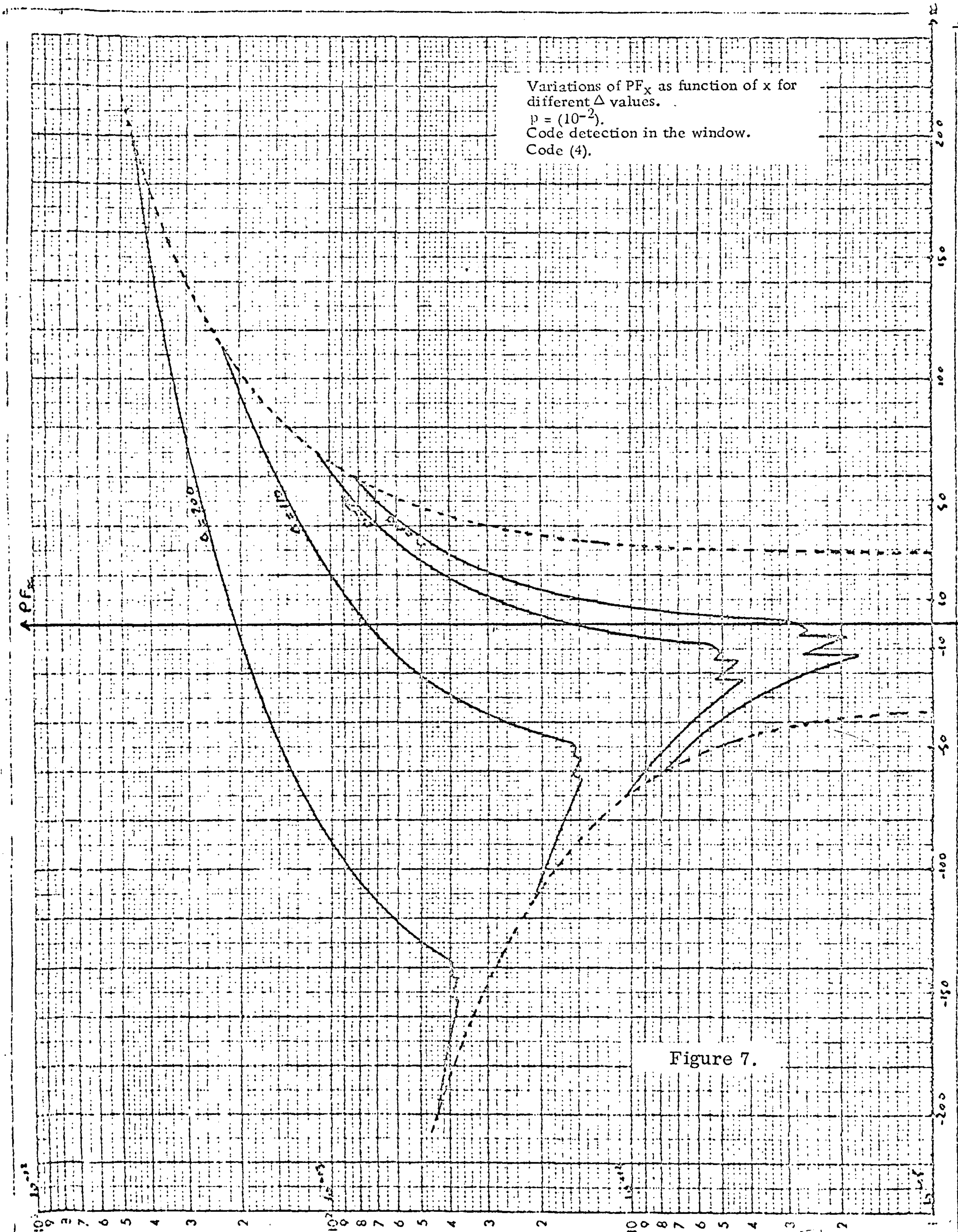
$$A_e = 1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e} C_N^j$$

$$\mathcal{H}(b,e) = \prod_{m=N-b-1}^{N-1} (1-Hm(e))$$

PV (x) is expressed more concisely by:

$$\boxed{PV(x) = \sum_{e=0}^{N} Pr(e)\, A_e^{\beta 1}\, A_{e-1}^{\beta 2}\, \mathcal{H}(b1,e)\, \mathcal{H}(b2,e-1)}$$

Several values of PV (x) as a function of x and of Δ are shown on the curves of Figs. 7 to 13 for different codes.



Variations of PF_x as function of x for different Δ values.
$p = (10^{-2})$.
Code detection in the window.
Code (4).

Figure 7.

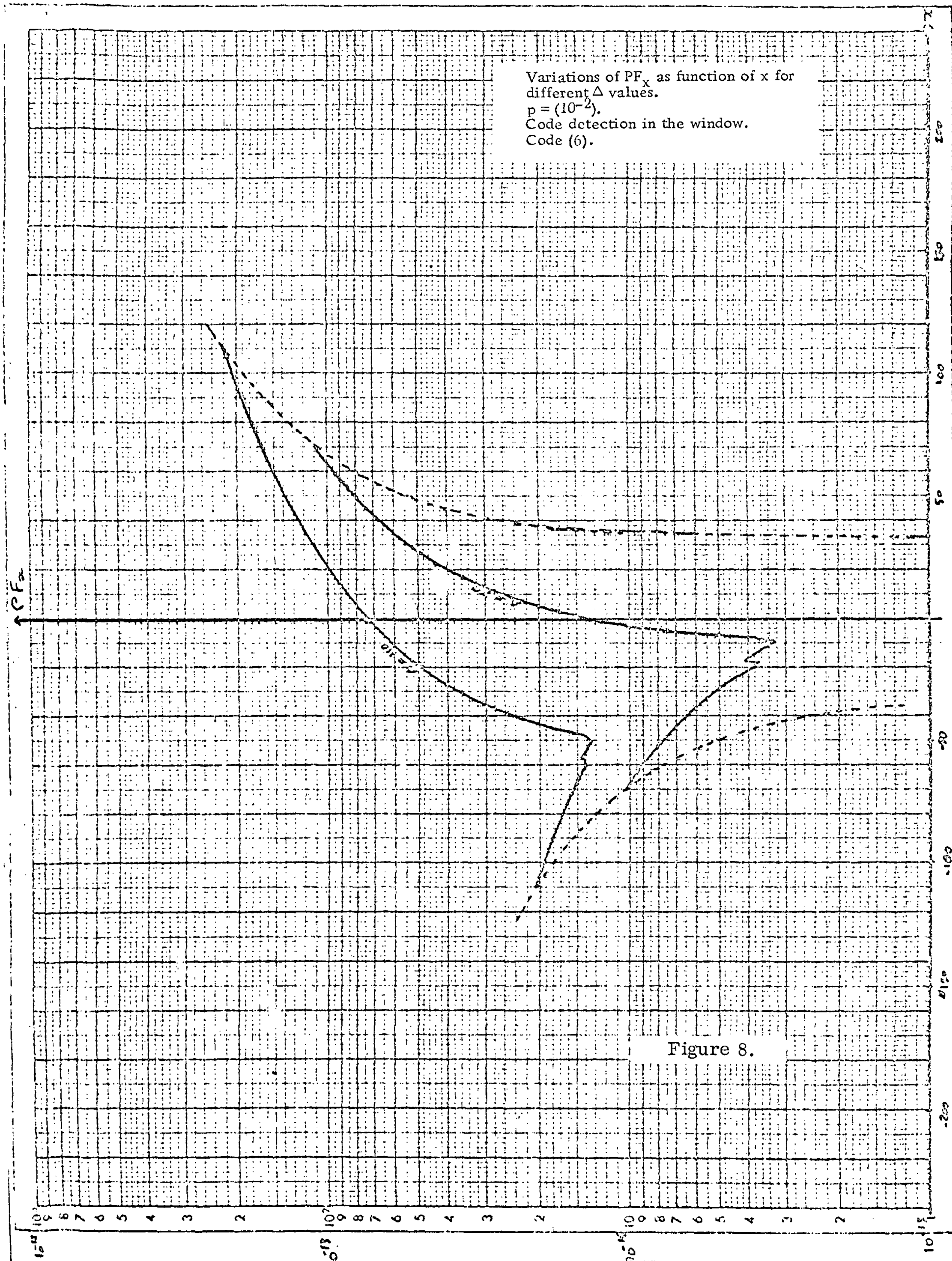

Variations of PF_x as function of x for different Δ values.
p = (10^{-2}).
Code detection in the window.
Code (6).
PF_x
x

Figure 8.

Centre National d'Etudes Spatiales	Variations of PF_x as function of x for different Δ values	$p = 10^{-6}$
		Code (1)

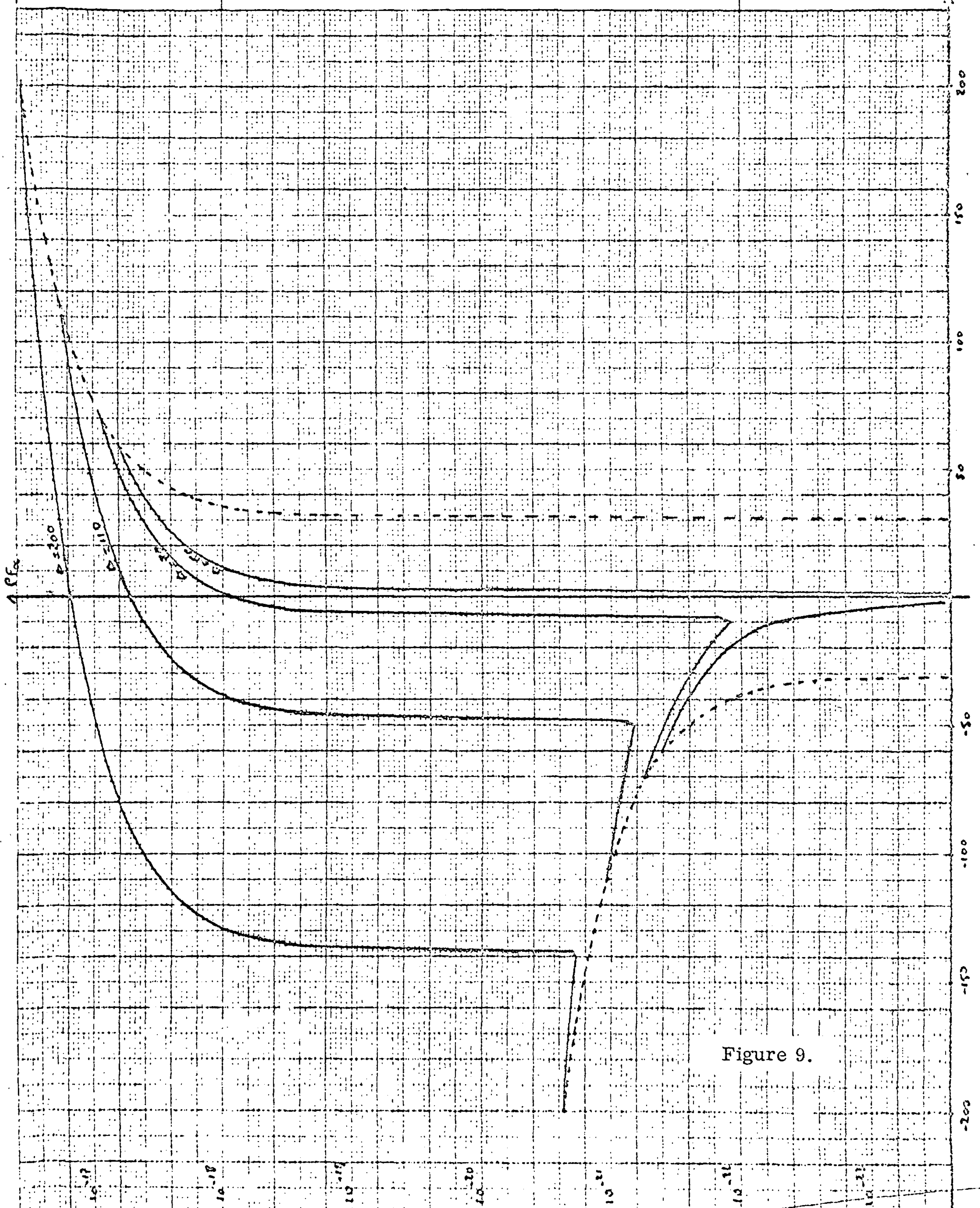


Figure 9.

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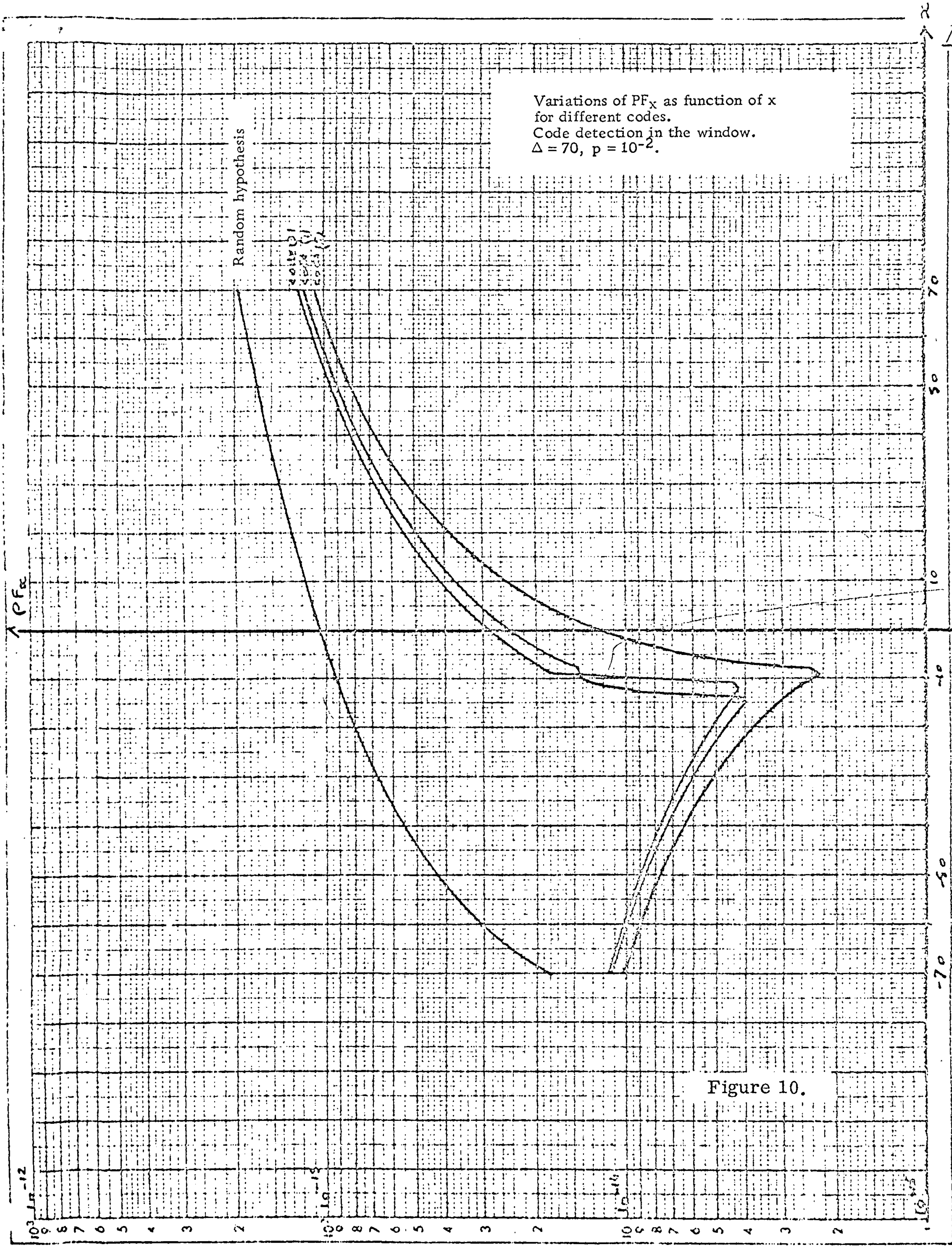


Variations of PF_x as function of x for different codes.
Code detection in the window.
$\Delta = 70$, $p = 10^{-2}$.

Figure 10.

Centre National d'Études Spatiales	Variations of PF_x as function of x for different codes	$p = 10^{-6}$
		$\Delta = 70$

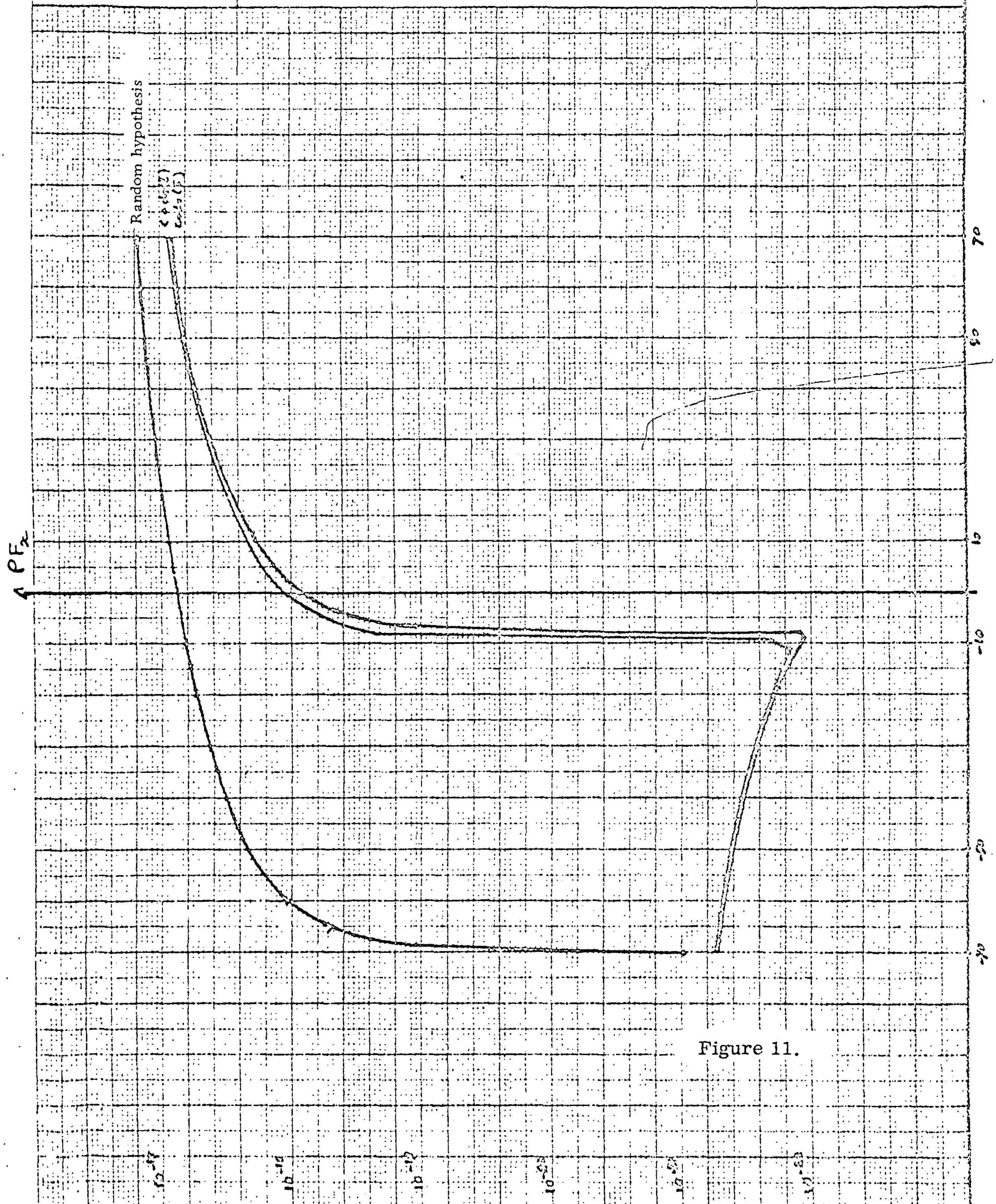


Figure 11.

Centre National d'Etudes Spatiales	Variations of PF_x as function of x for different Δ values. Code detection in the window.	$p = 10^{-5}$
		Code (5)

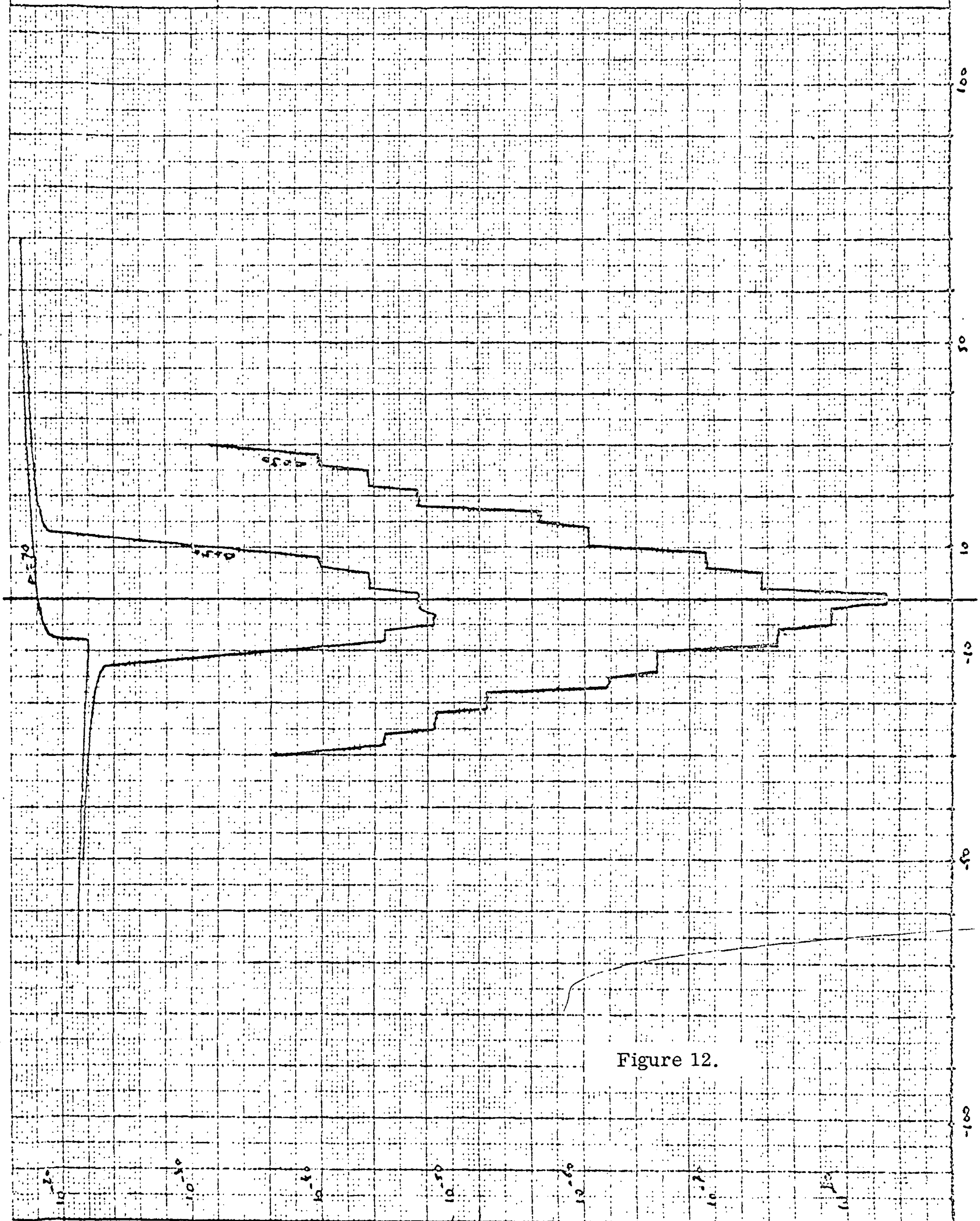


Figure 12.

Centre National d'Etudes Spatiales	Variations of PF_x as function of x for different Δ values. Code detection in the window.	$p = 10^{-2}$	/28
		Code (5).	

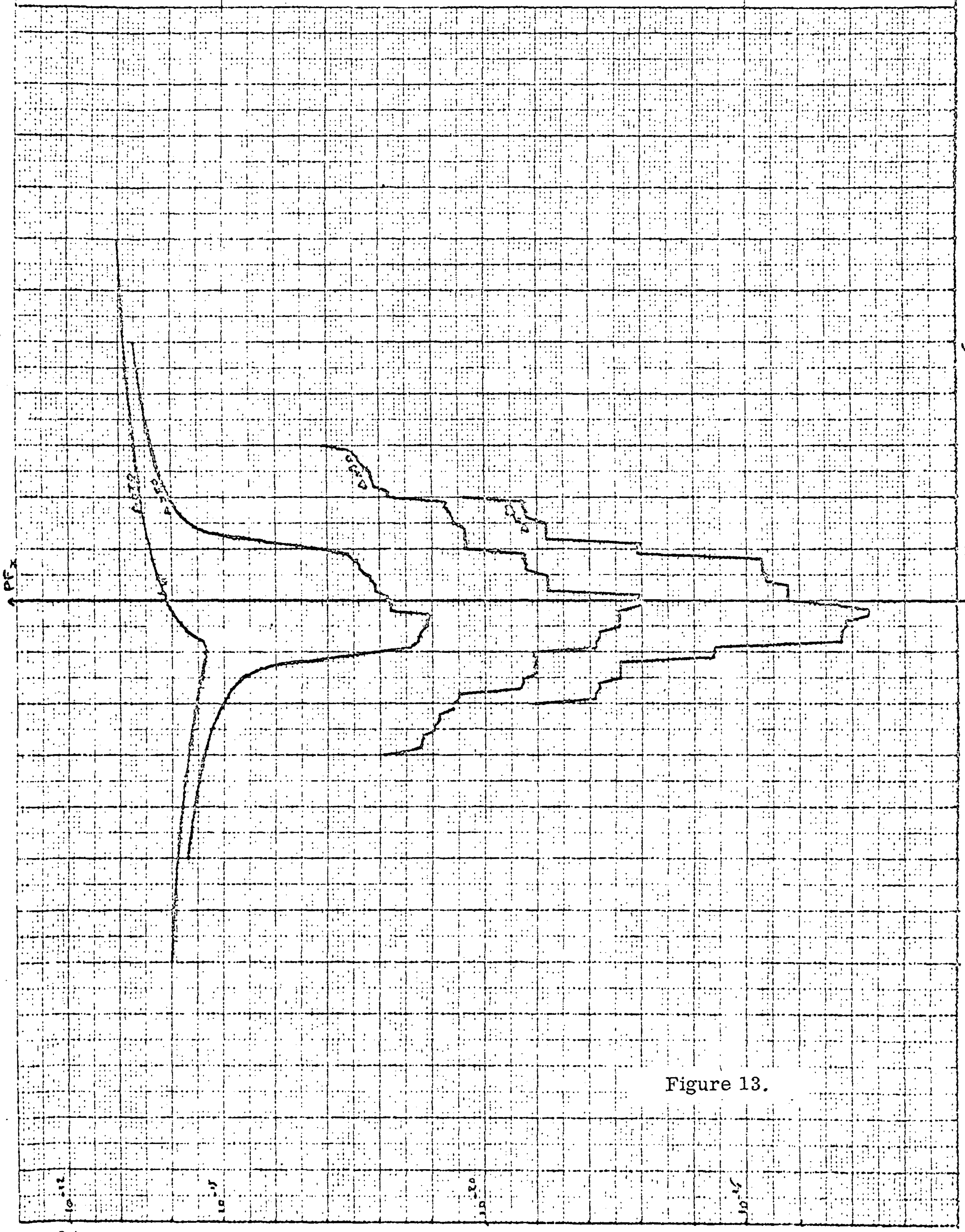


Figure 13.

The codes marked on the curves are as follows:

Codes 1, 2, 3, 4, 5 have the characteristic function $1 + x^5 + x^6$, i.e., the input of the shift register is the result of an "exclusive OR" between flip-flops 5 and 6. /29

For code 1 the initial state of the register is 111111.

For code 2 the initial state of the register is 011111.

For code 3 the initial state of the register is 000111.

For code 4 the initial state of the register is 100001.

For code 5 the initial state of the register is 100000.

Code 6 has the characteristic function $1 + x^2 + x^3 + x^5 + x^6$, i.e., the input of the shift register is the result of an "exclusive OR" between the flip-flops 2, 3, 5, 6. The initial state of the register is 111111.

It should be noted that the random hypothesis (all positions other than the code position have the random configurations) is permissible with respect to the real case where one must take account of the overlapping between the code and the adjacent positions.

Consequently, this hypothesis is valid and will be retained for the study of locking operation.

2) Locking operation

The calculations have been made and the curves plotted in the case of a non-alternated code. We shall see later what changes are imposed by an alternated code.

2.1) Case of an non-alternated code

We are interested in the case where the code has been correctly detected in the preceding cycle, i.e., the synchronization code of the cycle examined has been found in the window. Let x be its position ($x \in [-\Delta, +\Delta]$)

Let PVC(x) be the probability of correct detection of the code in the window, if the latter is at x, associated with a confirmation. Let PVI(x) be the probability of correct detection of the code in the window, if the latter is at x, associated with an invalidation. Let PFC(x) be the probability, if the code is at x, of making a false detection and of confirming this detection. Let PFI(x) be the probability, if the code is at x, of making a false detection and of invalidating this detection. Let E be the threshold value, a constant for the examination of the remainder of the cycle. /30

*Calculation of PVC(x)

If the received code exhibits e errors with respect to the expected code, the probability of correct detection in the window is

$$\Pr(e)\, A_e^{\Delta+x}\, A_e^{\Delta-x} \qquad (\S \text{II}. 1. 1)$$

The probability of confirming this detection is the probability that, at the ν positions remaining in the cycle before the window opens for the following cycle ($\nu = M - 2\Delta + 1$), there are ε errors or more ($\varepsilon = e/k$ or $\varepsilon = e - \delta$),

i.e.,

$$\left[1 - \left(\tfrac{1}{2}\right)^N \sum_{j=0}^{\varepsilon-1} C_N^j\right]^{\nu} = A_{\varepsilon-1}^{\nu}$$

whence

$$\boxed{PVC(x) = \sum_{e=0}^{N} \Pr(e)\, A_e^{\Delta+x}\, A_{e-1}^{\Delta-x}\, A_{\varepsilon-1}^{\nu}}$$

*Calculation of PVI(x)

For a like probability of detecting the code if the latter has e errors, the probability of an invalidation is the probability that, at the ν positions subsequently examined, there exists at least one for which the number of errors is less than e.

$$\boxed{PVI(x) = \sum_{e=0}^{N} \Pr(e)\, A_e^{\Delta+x}\, A_{e-1}^{\Delta-x} \left\{1 - A_{\varepsilon-1}^{\nu}\right\} = \sum_{e=0}^{N} \Pr(e)\, A_e^{\Delta+x}\, A_{e-1}^{\Delta-x} - PVC(x)}$$

*Calculation of PFC (x) /31

Let us denote by $PF_{y,e}(x)$ the probability that the position y ($y \in [-\Delta\ x - 1] \cup [x + 1, \Delta]$), exhibiting e errors with respect to the expected code, will be taken for the position of the cycle synchronization code, PFC(x) is then expressed by:

$$PFC(x) = \sum_{e=0}^{N} \left\{ \sum_{y=-\Delta}^{x-1} PF_{y,e}(x) + \sum_{y=x+1}^{\Delta} PF_{y,e}(x) \right\} A_{\varepsilon-1}^{\nu}$$

$PF_{y,\,e}(x)$ is the probability that all the positions preceding y have more than e errors and that the positions following y have e errors or more.

Si $y \in [-\Delta, x-1]$ $$PF_{y,e}(x) = \left(\tfrac{1}{2}\right)^N C_N^e \left(1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e} C_N^j\right)^{\Delta+y} \left(1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e-1} C_N^j\right)^{\Delta-y-1} \sum_{j=e}^{N} C_N^j p^j (1-p)^{N-j}$$

$$PF_{y,e}(x) = \left(\tfrac{1}{2}\right)^N C_N^e A_e^{\Delta+y} A_{e-1}^{\Delta-y-1} \sum_{j=e}^{N} Pr(j)$$

Si $y \in [x+1, \Delta]$ $$PF_{y,e}(x) = \left(\tfrac{1}{2}\right)^N C_N^e \left(1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e} C_N^j\right)^{\Delta+y-1} \left(1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e-1} C_N^j\right)^{\Delta-y} \sum_{j=e+1}^{N} C_N^j p^j (1-p)^{N-j}$$

$$PF_{y,e}(x) = \left(\tfrac{1}{2}\right)^N C_N^e A_e^{\Delta+y-1} A_{e-1}^{\Delta-y} \sum_{j=e+1}^{N} Pr(j)$$

$$PFC(x) = \sum_{e=0}^{N} \left(\tfrac{1}{2}\right)^N C_N^e A_{\varepsilon-1}^{\partial} \left\{ \sum_{y=-\Delta}^{x-1} A_e^{\Delta+y} A_{e-1}^{\Delta-y-1} \sum_{j=e}^{N} Pr(j) + \sum_{y=x+1}^{\Delta} A_e^{\Delta+y-1} A_{e-1}^{\Delta-y} \sum_{j=e+1}^{N} Pr(j) \right\}$$

$$= \sum_{e=0}^{N} \left(\tfrac{1}{2}\right)^N C_N^e A_{\varepsilon-1}^{\partial} \left\{ \sum_{j=e}^{N} Pr(j) \sum_{y=-\Delta}^{x-1} A_e^{\Delta+y} A_{e-1}^{\Delta-y-1} + \sum_{j=e+1}^{N} Pr(j) \sum_{y=x+1}^{\Delta} A_e^{\Delta+y-1} A_{e-1}^{\Delta-y} \right\}$$

$$\sum_{y=-\Delta}^{x-1} A_e^{\Delta+y} A_{e-1}^{\Delta-y-1} = A_{e-1}^{2\Delta-1} \frac{1-(A_e/A_{e-1})^{\Delta+x}}{1-A_e/A_{e-1}} = A_{e-1}^{2\Delta-1} \frac{A_{e-1}^{\Delta+x} - A_e^{\Delta+x}}{A_{e-1}^{\Delta+x}} \; \frac{A_{e-1}}{A_{e-1}-A_e}$$

$$= A_{e-1}^{\Delta-x} \frac{A_{e-1}^{\Delta+x} - A_e^{\Delta+x}}{\left(\tfrac{1}{2}\right)^N C_N^e} = \frac{A_{e-1}^{2\Delta} - A_e^{\Delta+x} A_{e-1}^{\Delta-x}}{\left(\tfrac{1}{2}\right)^N C_N^e}$$

$$\sum_{y=x+1}^{\Delta} A_e^{\Delta+y-1} A_{e-1}^{\Delta-y} = A_e^{\Delta+x} A_{e-1}^{\Delta-x-1} \frac{1-(A_e/A_{e-1})^{\Delta-x}}{1-A_e/A_{e-1}} = A_e^{\Delta+x} A_{e-1}^{\Delta-x-1} \frac{A_{e-1}^{\Delta-x} - A_e^{\Delta-x}}{A_{e-1}^{\Delta-x}} \; \frac{A_{e-1}}{A_{e-1}-A_e}$$

$$\sum_{y=x+1}^{\Delta} A_e^{\Delta+y-1} A_{e-1}^{\Delta-y} = A_e^{\Delta+x} \frac{A_{e-1}^{\Delta-x} - A_e^{\Delta-x}}{\left(\tfrac{1}{2}\right)^N C_N^e} = \frac{A_e^{\Delta+x} A_{e-1}^{\Delta-x} - A_e^{2\Delta}}{\left(\tfrac{1}{2}\right)^N C_N^e}$$ /32

$$PFC(x) = \sum_{e=0}^{N} A_{\varepsilon-1}^{\partial} \left\{ \sum_{j=e}^{N} Pr(j) \left(A_{e-1}^{2\Delta} - A_e^{\Delta+x} A_{e-1}^{\Delta-x}\right) + \sum_{j=e+1}^{N} Pr(j) \left(A_e^{\Delta+x} A_{e-1}^{\Delta-x} - A_e^{2\Delta}\right) \right\}$$

$$\boxed{PFC(x) = \sum_{e=0}^{N} A_{\varepsilon-1}^{\partial} \left\{ A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) - Pr(e)\, A_e^{\Delta+x} A_{e-1}^{\Delta-x} \right\}}$$

PFC(x) can also be written as:

$$PFC(x) = \sum_{e=0}^{N} A_{\varepsilon-1}^{\partial} \left\{ A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) \right\} - PVC(x)$$

when e = o, ε = o and $A_{\varepsilon - 1} = A_{e - 1} = 1$

$$PFC(x) = 1 - A_0^{2\Delta} \sum_{j=1}^{N} Pr(j) + \sum_{e=1}^{N} A_{\varepsilon-1}^{\partial} \left\{ A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) \right\} - PVC(x)$$

$$\boxed{\begin{gathered} PFC(x) = 1 - PVC(x) - \lambda \\ \text{avec : } \lambda = A_0^{2\Delta} \sum_{j=1}^{N} Pr(j) - \sum_{e=1}^{N} A_{\varepsilon-1}^{\partial} \left\{ A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) \right\} \end{gathered}}$$

<u>*Calculation of PFI (x)</u>

By analogy with PFC(x), PFI(x) is expressed by:

$$\boxed{PFI(x) = \sum_{e=0}^{N} \left(1 - A_{\varepsilon-1}^{\partial}\right) \left\{ A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) - Pr(e)\, A_e^{\Delta+x} A_{e-1}^{\Delta-x} \right\}}$$

PFI(x) can also be written as:

$$PFI(x) = \sum_{e=0}^{N} \left(1 - A_{\varepsilon-1}^{\partial}\right) \left\{ A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) \right\} - PVI(x)$$

$$PFI(x) = \sum_{e=0}^{N} A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - \sum_{e=0}^{N} A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) - \sum_{e=0}^{N} A_{\varepsilon-1}^{\partial} \left\{ A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) \right\} - PVI(x)$$

$$PFI(x) = 1 + \sum_{e=1}^{N} A_{e-1}^{2\Delta} \sum_{j=e}^{N} Pr(j) - \sum_{e=0}^{N} A_e^{2\Delta} \sum_{j=e+1}^{N} Pr(j) - 1 + \lambda - PVI(x) = \lambda - PVI(x)$$

$$\boxed{PFI(x) = \lambda - PVI(x)}$$

The values of PVC(x), PVI(x) and PFC(x) are indicated on the curves of Figs. 14 to 19. The PFI(x) values are not listed since the calculation accuracy is inadequate. One can simply say that for $p = 10^{-2}$, PFI(x) is less than 10^{-18} and that for $p = 10^{-6}$, PFI(x) is less than 10^{-27}. /33

In the case of a correct detection, the probability of a confirmation must therefore be as high as possible; the probability of an invalidation must then be as low as possible. In the case of an incorrect detection, the opposite is true. Since these two requirements are contradictory, a compromise is found.

From the curves it seems that k = 2 would be a correct value, where $\delta = 1$, 2, or 3. Since division by 2 is simpler to do than a subtraction, and it suffices to accomplish a shift, this is the solution that is retained.

2.2) <u>The case of an alternated code</u>

In the window, each position is compared to the expected code, but outside the window the comparison is made with respect to the code and its complement. If there are e' errors with respect to the code, there are N - e' errors with respect to the code complement. Consequently, outside the window the number or errors after the comparison is less than N/2. It will then only be possible to have confirmation if the detection in the window is made with less than N/2 errors ($e \leq \frac{N-1}{2}$ if N is odd).

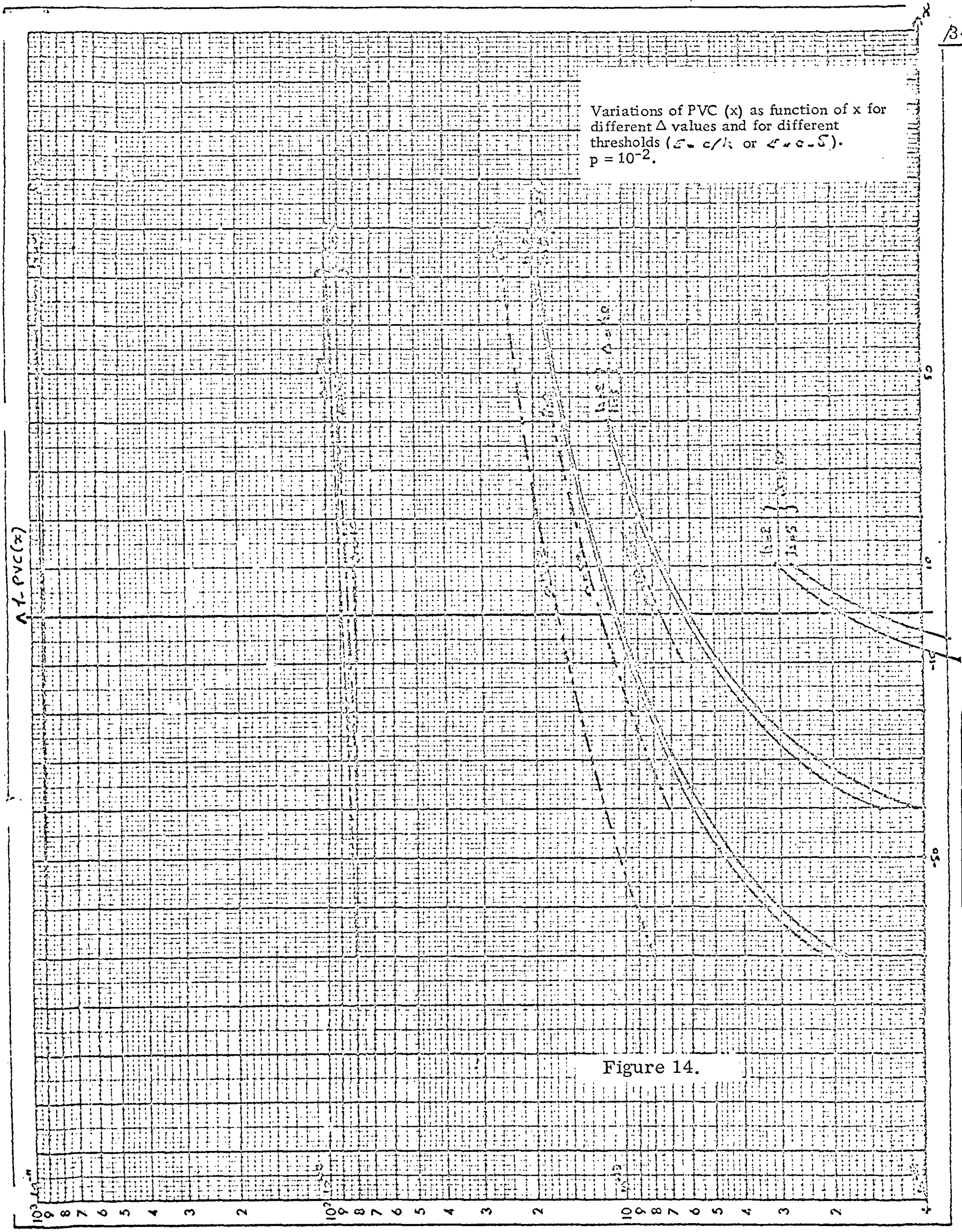
$$PVC(x) = \sum_{e=0}^{(N-1)/2} C_N^e\, p^e (1-p)^{N-e} \left(1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e} C_N^j\right)^{\delta+x} \left(1-\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{e-1} C_N^j\right)^{\delta-x} \left(\left(\tfrac{1}{2}\right)^N \sum_{j=\varepsilon}^{N-\varepsilon} C_N^j\right)^{\delta}$$

Let $\quad B_{\varepsilon-1} = \left(\tfrac{1}{2}\right)^N \sum_{j=\varepsilon}^{N-\varepsilon} C_N^j = 1 - 2\left(\tfrac{1}{2}\right)^N \sum_{j=0}^{\varepsilon-1} C_N^j = 2A_{\varepsilon-1} - 1$

$$PVC(x) = \sum_{e=0}^{(N-1)/2} Pr(e)\, A_e^{\delta+x}\, A_{e-1}^{\delta-x}\, B_{\varepsilon-1}^{\delta}$$ /40

$$PVI(x) = \sum_{e=(N+1)/2}^{N} Pr(e) \cdot A_e^{\delta+x}\, A_{e-1}^{\delta-x} + \sum_{e=0}^{(N-1)/2} Pr(e)\, A_e^{\delta+x}\, A_{e-1}^{\delta-x} \left\{1 - B_{\varepsilon-1}^{\delta}\right\}$$

$$PFC(x) = \sum_{e=0}^{(N-1)/2} B_{\varepsilon-1}^{\delta} \left\{A_{e-1}^{2\delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\delta} \sum_{j=e+1}^{N} Pr(j) - Pr(e)\, A_e^{\delta+x}\, A_{e-1}^{\delta-x}\right\}$$



Variations of PVC (x) as function of x for different Δ values and for different thresholds ($\xi = c/k$ or $\xi = c - \delta$).
$p = 10^{-2}$.

Figure 14.

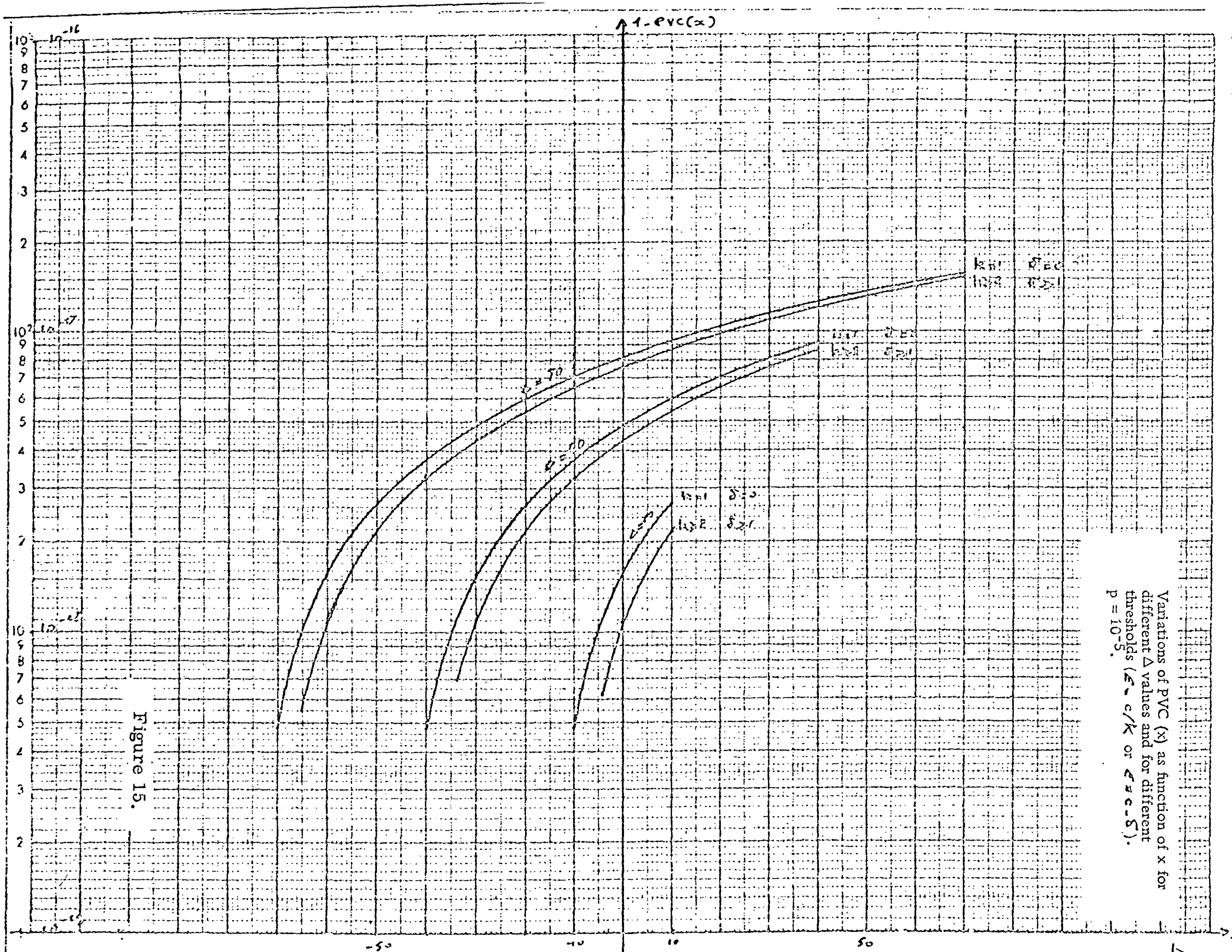


Figure 15.

Variations of PVC (x) as function of x for different Δ values and for different thresholds ($\varepsilon = c/k$ or $\varepsilon = c-\delta$). $p = 10^{-5}$.

Variations of PVI (x) as function of x for different Δ values and for different thresholds ($\varepsilon = c/k$ or $\varepsilon = c - \delta$). True detection follows an invalidation.

$p = 10^{-2}$

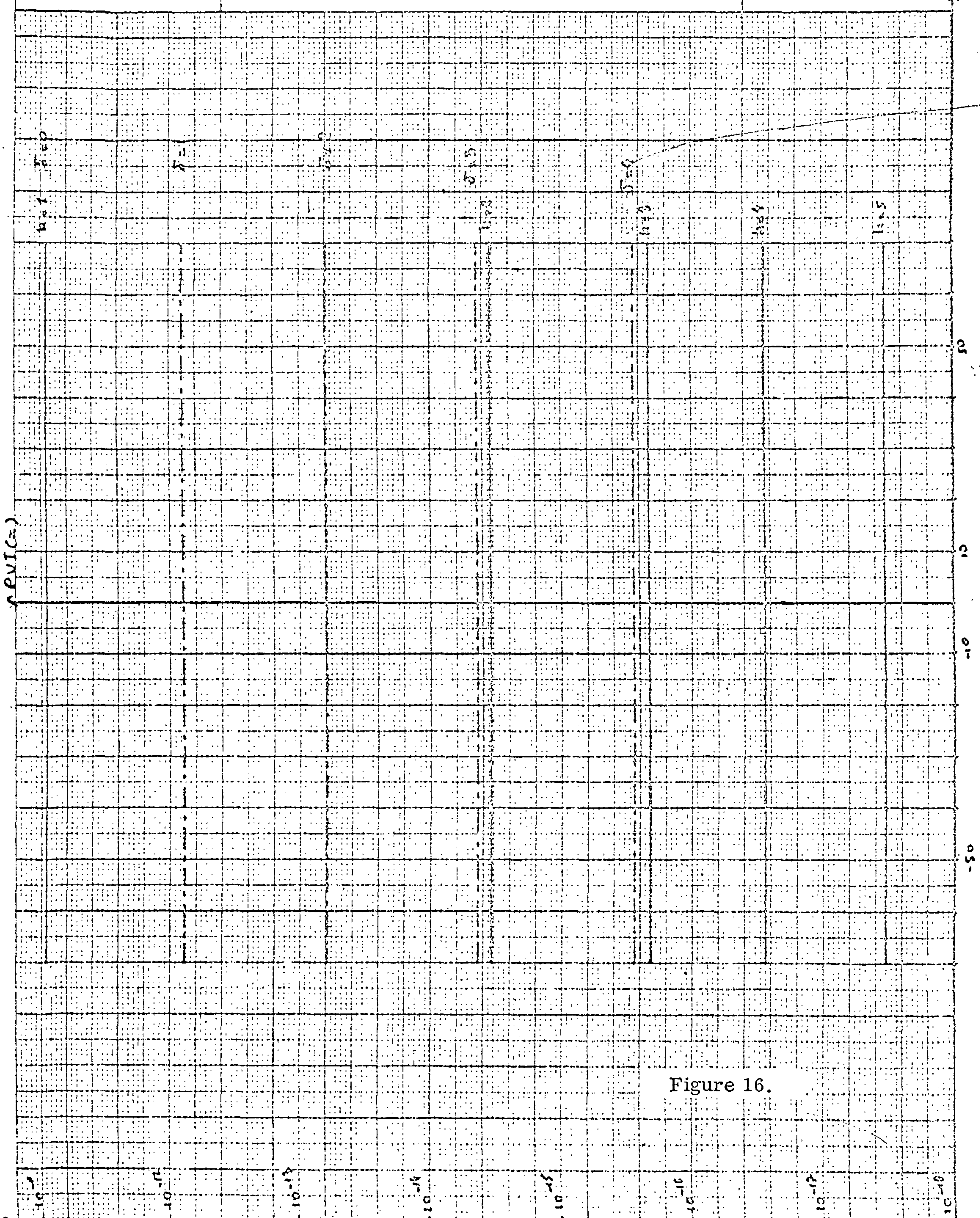


Figure 16.

Variations of PVI (x) as function of x for different Δ values and for different thresholds ($\xi = c/k$ or $\xi = c - \delta$). True detection follows an invalidation.

$p = 10^{-6}$.

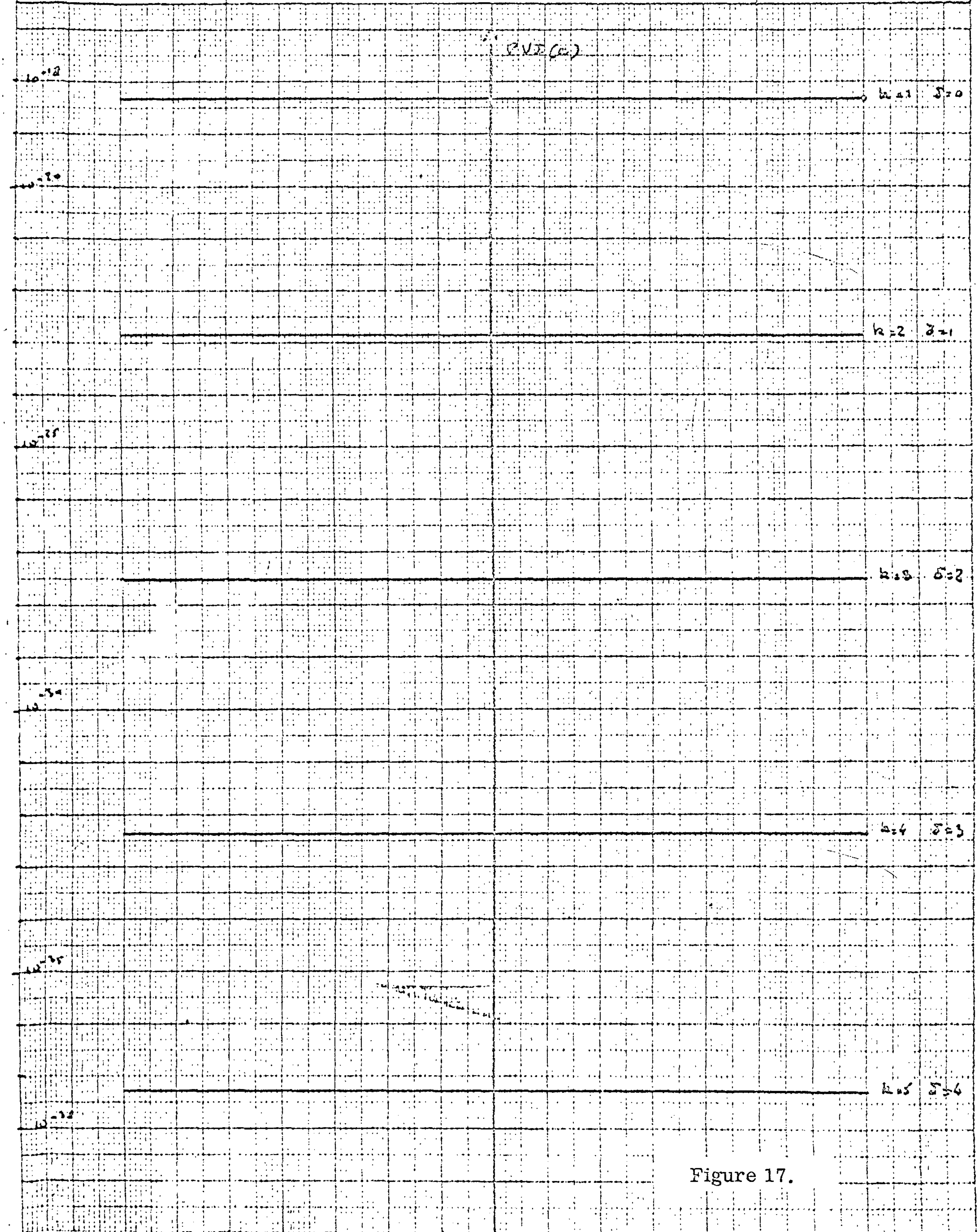
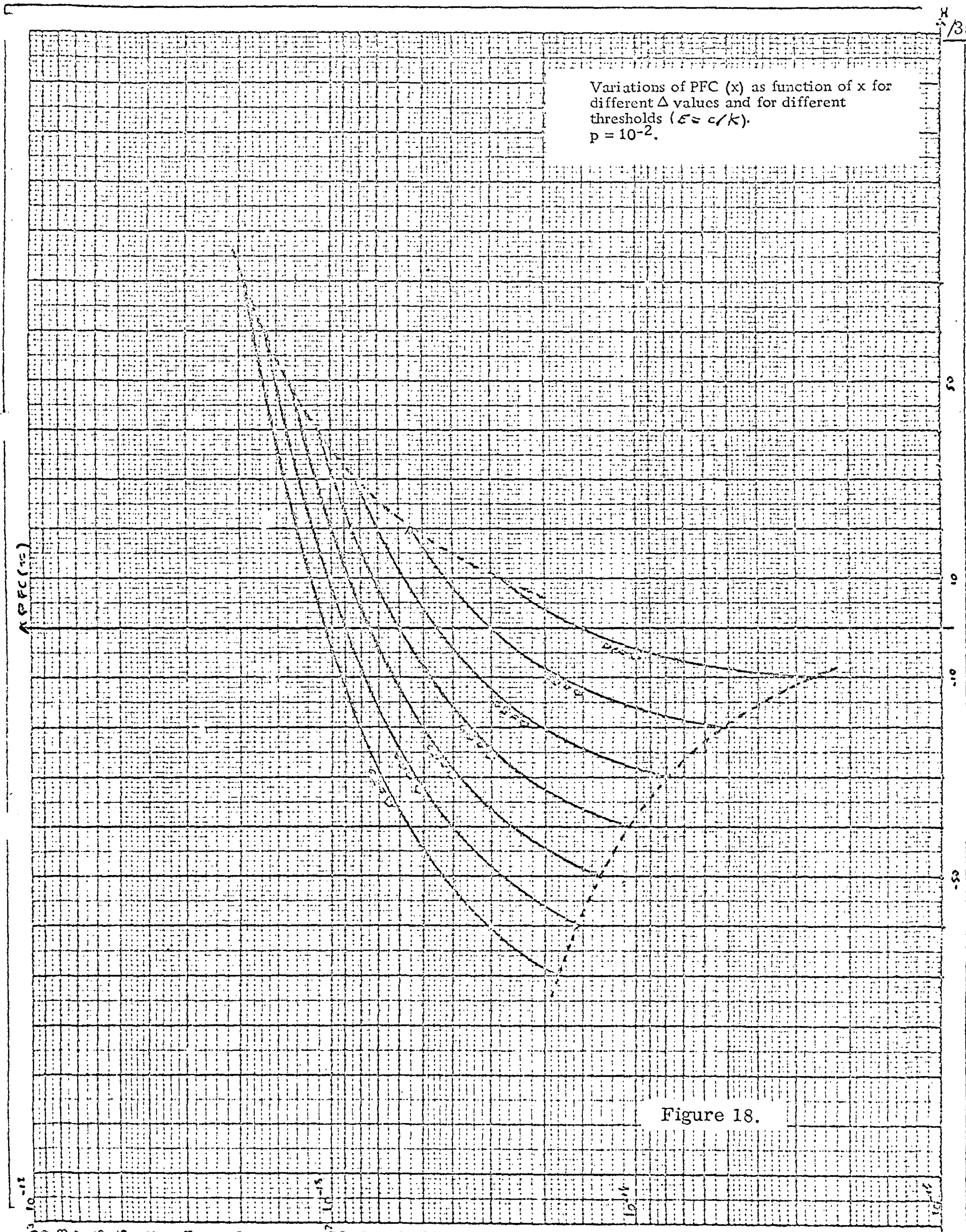


Figure 17.



Variations of PFC (x) as function of x for different Δ values and for different thresholds ($\varepsilon = c/k$).
$p = 10^{-2}$.

Figure 18.

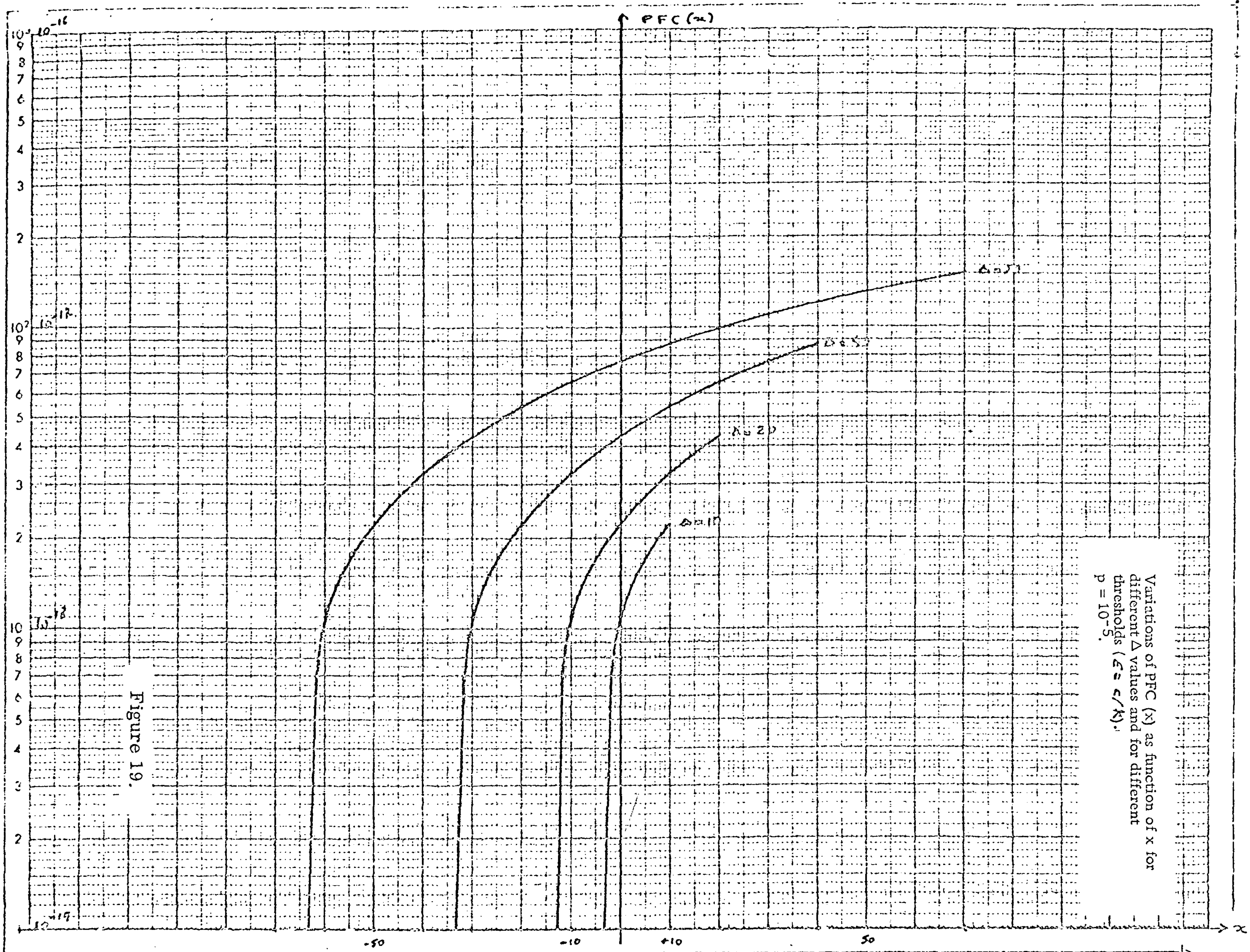


Figure 19.

Variations of PFC (x) as function of x for different Δ values and for different thresholds ($\mathcal{E} = c/k$). $p = 10^{-5}$.

$$PFI(x) = \sum_{e=(N+1)/2}^{N} \left\{ A_{e-1}^{2\delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\delta} \sum_{j=e+1}^{N} Pr(j) - Pr(e) A_e^{\Delta+x} A_{e-1}^{\Delta-x} \right\} + \sum_{e=0}^{(N-1)/2} \left\{ 1 - B_{\varepsilon-1}^{\nu} \right\} \left\{ A_{e-1}^{2\delta} \sum_{j=e}^{N} Pr(j) - A_e^{2\delta} \sum_{j=e+1}^{N} Pr(j) - Pr(e) A_e^{\Delta+x} A_{e-1}^{\Delta-x} \right\}$$

2.3) Comparison between alternated or non-alternated code

$$* \left[1 - PVC(x) \right]_{na} = \sum_{e=0}^{N} Pr(e) \left\{ 1 - A_e^{\Delta+x} A_{e-1}^{\Delta-x} A_{\varepsilon-1}^{\nu} \right\}$$

$$\left[1 - PVC(x) \right]_{a} = \sum_{e=(N+1)/2}^{N} Pr(e) + \sum_{e=0}^{(N-1)/2} Pr(e) \left\{ 1 - A_e^{\Delta+x} A_{e-1}^{\Delta-x} B_{\varepsilon-1}^{\nu} \right\}$$

The calculation shows that the terms beyond N/2 are negligible

$$\left[1 - PVC(x) \right]_{na} \# \sum_{e=0}^{N/2} Pr(e) \left\{ 1 - A_e^{\Delta+x} A_{e-1}^{\Delta-x} A_{\varepsilon-1}^{\nu} \right\}$$

$$\left[1 - PVC(x) \right]_{a} \# \sum_{e=0}^{N/2} Pr(e) \left\{ 1 - A_e^{\Delta+x} A_{e-1}^{\Delta-x} B_{\varepsilon-1}^{\nu} \right\} > \left[1 - PVC(x) \right]_{na}$$

A_e is of the form $1 - a_e$, $a_e \ll 1$

B_e is of the form $1 - 2a_e$, $a_e \ll 1$

For the values of e which are to be taken into consideration in the calculation, one also has: /41

$$(\Delta + x)\, a_e \ll 1 \qquad (\Delta - x)\, a_e \ll 1 \qquad \nu\, a_e \ll 1$$

Consequently

$$\left[1 - PVC(x) \right]_{na} \# \sum_{e=0}^{N/2} Pr(e) \left\{ (\Delta + x) a_e + (\Delta - x) a_e + \nu\, a_{\varepsilon-1} \right\}$$

$$\left[1 - PVC(x) \right] a \# \sum_{e=0}^{N/2} Pr(e) \left\{ (\Delta + x) a_e + (\Delta - x) a_e + 2\nu\, a_{\varepsilon-1} \right\}$$

If $\varepsilon = e$, i.e., $k = 1$ or $\delta = 0$ $[1 - PVC(x)]\, a = 2\, [1 - PVC(x)]\, na$

If $\varepsilon = e/2$, i.e., $k = 2$ [1 - PVC(x)] a $\not\!\!/\!\!/$ [1 - PVC(x)] na + $4 10^{-15}$

for $p = 10^{-2}$

then [1 - PVC(x)] a $\not\!\!/\!\!/$ [1 - PVC(x)] na if $k \geq 3$ for $p = 10^{-2}$, if $k \geq 2$ for $p = 10^{-6}$.

* [PVI (x)] a $\not\!\!/\!\!/$ 2 [PVI(x)] na regardless of values of p, Δ, k or δ.
* PFC (x) remains unchanged.

The conclusions reached in the case of a non-alternated code are then valid for an alternated code, that is:

$$\boxed{\varepsilon = e/2}$$

3) Average time of locking operation

The case of a correct detection of the code in the window.

The probability P(j) of remaining in the locking phase during j cycles at one correct position is the probability of having a confirmation during j cycles and an invalidation at the j + 1 cycle.

Thus $P(j) = PVC^j \; (PVI + PFI)$

with $PVC = \sum_{x=-\Delta}^{+\Delta} \delta(x) \; PVC(x)$ where δ (x) is the distribution probability of the code in the window

$$PVI = \sum_{x=-\Delta}^{+\Delta} \delta(x) \; PVI \; (x)$$ /42

$$PFI = \sum_{x=-\Delta}^{+\Delta} \delta(x) \; PFI \; (x)$$

The average value of j is given by:

$$\langle j \rangle = \frac{\sum_{j=1}^{\infty} j \; PVC^j \; (PVI + PFI)}{\sum_{j=0}^{\infty} PVC^j \; (PVI + PFI)} = \frac{PVC \sum_{j=1}^{\infty} j \; PVC^{j-1}}{\sum_{j=0}^{\infty} PVC^j}$$

$$\langle j \rangle = \frac{PVC \sum_{j=1}^{\infty} \frac{d}{dPVC} PVC^j}{1/(1-PVC)} = PVC \; (1-PVC) \frac{d}{dPVC} \left(\frac{PVC}{1-PVC}\right)$$

$$= PVC \; (1-PVC) \frac{1}{(1-PVC)^2}$$

$$\boxed{\langle j \rangle = \frac{PVC}{1-PVC}}$$

For a uniform distribution of the code in the window PVC is essentially equal to PVC(0).

$$p = 10^{-2} \qquad k = 1 \qquad \langle j \rangle \simeq 5{,}5\ 10^{9}$$

$$p = 10^{-6} \qquad k = 1 \quad \Delta = 10 \quad \langle j \rangle \simeq 3{,}1\ 10^{17}$$

$$p = 10^{-2} \qquad k = 2 \quad \Delta = 10 \quad \langle j \rangle \simeq 4{,}4\ 10^{13}$$

$$p = 10^{-6} \qquad k = 2 \quad \Delta = 10 \quad \langle j \rangle \simeq 9\ 10^{+17}$$

C. SUBCYCLE SYNCHRONIZATION /43

When the cycle synchronization consists of detecting the position of a given bit configuration, the subcycle synchronization, in the case of a synchronization by cycle counting, consists of determining the bit configuration of a given position.

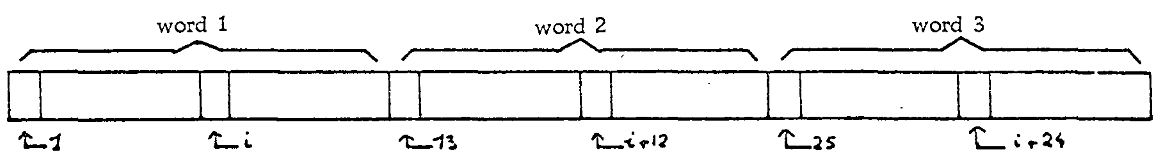
This configuration determination is accomplished in two steps:

- a first reduction in the number of errors by majority decision,

- properly stated synchrnoization.

I. Detection of the identification word by majority decision

The information received concerning the cycle counting consists of three words of 12 theoretically identical bits. As a result of transmission errors, certain bits are erroneous:



The bits i, i + 12, i + 24, occupying the same position in the words 1, 2 and 3, are compared. The value found two or three times among these 3 bits will be the value assigned to bit i of the detected word. This value will be exact if there is no more than 1 error among the 3 bits received.

If p denotes the bit error probability up to the detection, and p_b is the bit error probability after detection,

$$P_b = p^3 + 3p^2(1-p) = 3p^2 - 2p^3$$

$$\boxed{\begin{array}{lcl} & & Pb = 3p^2 - 2p^3 \\ p=10^{-2} & \Longrightarrow & P_b = 2{,}98\ 10^{-4} \\ p=10^{-6} & \Longrightarrow & P_b = 3\ 10^{-12} \end{array}}$$

If p_0 denotes the probability that the detected word (ID word) is correct, since this latter contains 12 bits with an error probability p_b, /44

$$Po = (1 - p_b)^{12}$$

The probability that an ID word is incorrect is then $1 - p_0$, and the average number of incorrect words in L is equal to $L(1 - p_0)$

$$L\ (1-Po) = 1 \Longleftrightarrow L = \frac{1}{1-P_o} = \frac{1}{1-(1-p_b)^{12}}$$

$$\begin{array}{ll} p = 10^{-2} & L \# 280 \\ p = 10^{-6} & L \# 2.78 \times 10^{10} \end{array}$$

In other words, for $p = 10^{-2}$ there is, on the average, 1 false ID word in 280; for $p = 10^{-6}$ there is, on the average, 1 false ID word in 2.78×10^{10}.

II. Properly stated synchronization

The different synchronization phases are shown in Fig. 20.

1) Search Phase

Given that it is impossible to form any hypothesis concerning the value of the ID word, the word value that is received is assumed to be without error and is transferred into the cycle counter.

2) Control Phase

In the next cycle the cycle counter has been incremented by one unit and its value is compared to the received ID word. There is confirmation if the number of errors is zero, otherwise there is invalidation. A single invalidation causes a return to the search phase; J successive confirmations lead to the locking phase.

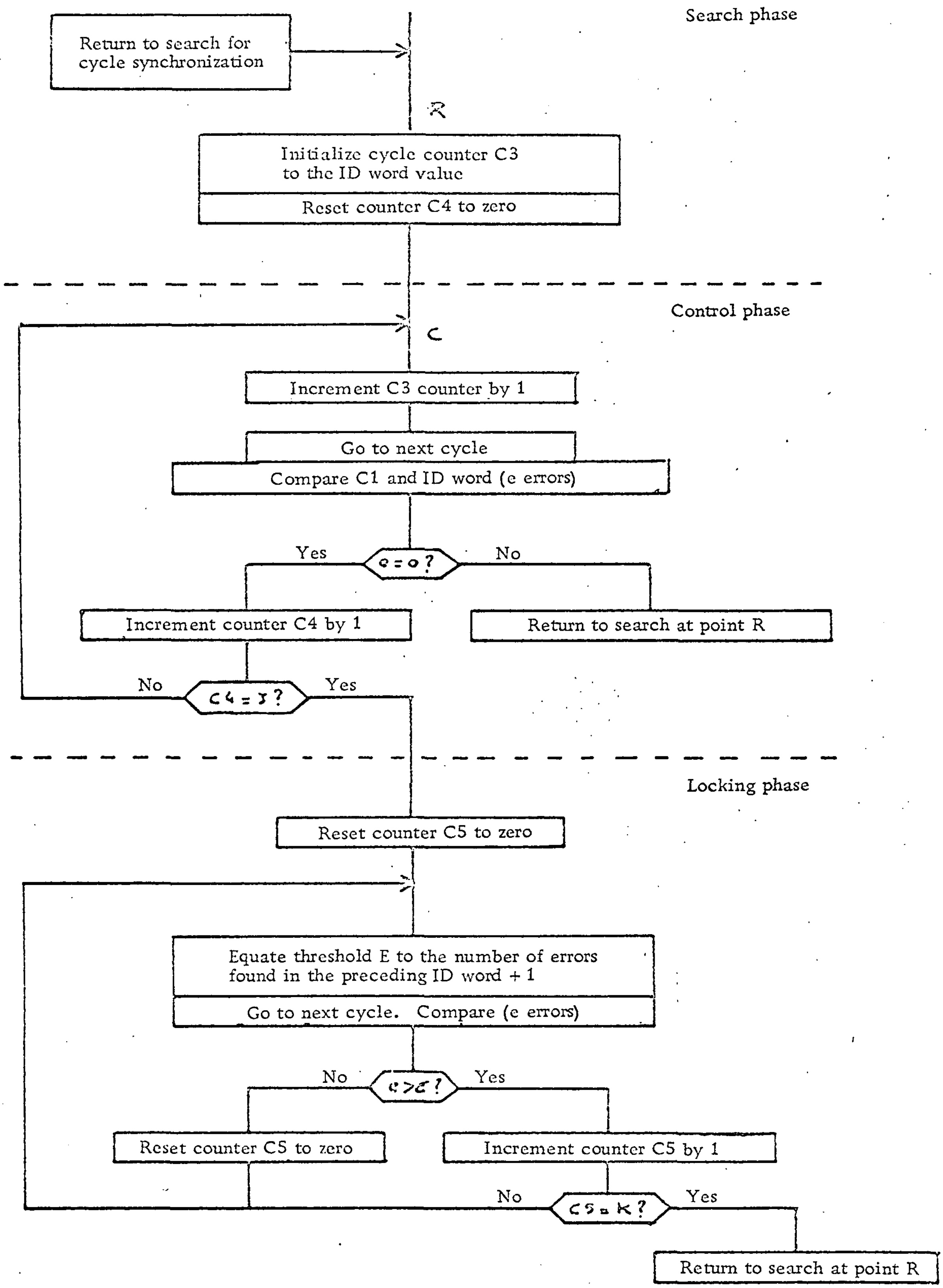


Figure 20.

Subcycle synchronization

3) <u>Locking Phase</u> /45

The locking phase is based on the variable threshold principle.

For a given cycle the threshold is fixed to the value of the number of errors in the preceding cycle, incremented by one unit. There is a confirmation if the number of errors is less than or equal to the threshold, otherwise an invalidation. K successive invalidations cause a return to the search phase.

The values of J and K are not fixed to an actual time but certainly have rather low values (<4).

APPENDIX

Justification of the Code Choice

/47

We recall that a PN code of length $n = 2^k - 1$ is obtained by means of k flip-flops mounted in a shift register, with input to the register being the result of an "exclusive OR" between several of the flip-flops. On the other hand, a code is described by its "characteristic polynomial" of the form $1 + \Sigma x^i$, with the i values corresponding to the flip-flops entering into the reaction.

For example, the polynomial $1 + x^5 + x^6$ represents an "exclusive OR" between flip-flops 5 and 6 (1 is the register input, k is the output).

A given characteristic polynomial provides the k circular permutations of one code, following the initial state of the flip-flops.

For a 63-bit code there are 6 characteristic polynomials:

(1) $1 + x^5 + x^6$ (3) $1+x^2+x^3+x^5+x^6$ (5) $1+x+x^2+x^5+x^6$

(2) $1 + x + x^6$ (4) $1+x+x^3+x^4+x^6$ (6) $1+x+x^4+x^5+x^6$

The polynomial (2) generates the mirror codes of the codes given by the polynomial (1). The same is true for the codes (3) and (4), (5) and (6). A code and its mirror have exactly the same synchronization properties; there are $3 \times 63 = 189$ possible codes.

The study of the Search phase has been based on the 63 codes generated by the polynomial (1). The results have shown that the best of these codes is that which begins with 5 "0" and ends with 6 "1". The Locking phase study has yielded the same results. /48

As a result, it is for a position that slightly overlaps the code that the probability of resemblance with the latter is greater. Thus the best codes are those which begin with the numerals "1" and terminate with the numerals "0" or vice versa.

Neither polynomial (3) nor (5) permits such codes to be generated. Consequently two possibilities remain:

Polynomial (1) in the initial state 100000, which gives 000001....0111111

Polynomial (2) in the initial state 111111, which gives 1111110...1000000

This is the second solution that is retained.

For the 127 code an extension of the foregoing results leads to choosing the code beginning with 7 "1" and ending with 6 "0". This code is obtained by the polynomial $1 + x + x^7$, with the initial state of the flip-flops being 1111111.

REFERENCES /49

1. Hamming, R. W. Error Detecting and Error Correcting Codes, BSTJ, XXVI, April 1950, p. 147-160.
2. Darche, N. Meteosat - PCM Transmission of Images - Format Definition, Report No. 533/CB/ES/E.
3. Boyer, M. Establishment of a Boundary Synchronization, Letter.

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